॥आचार्य सारथी॥
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JEE Main Free Mock 2

JEE (Main) 2027 Pattern · Full Syllabus · Free mock test

Date	Practise any day (3 hours)	**Duration**	3 Hours (180 minutes)
Maximum Marks	300 (100 per subject)	**Questions**	75 (25 per subject)

Student Name		**Roll No. / ID**	
School / Class		**Parent's sign**	

Marking scheme (each subject)

Section	Question type	Qs	Marks	Max
A	Single correct option (MCQ)	20	+4 correct, −1 wrong, 0 unattempted	80
B	Numerical value (integer)	5	+4 correct, −1 wrong, 0 unattempted	20
			Total per subject	**100**

Instructions

1. The paper has three parts: Part A Physics (Q.1–25), Part B Chemistry (Q.26–50) and Part C Mathematics (Q.51–75). Each part has Section A (20 MCQs) and Section B (5 numerical-value questions).
2. All questions are compulsory. Each MCQ has exactly one correct option.
3. The answer to every numerical-value question is an integer; it may be negative. Do not enter units.
4. Unless stated otherwise take $g = 10\ \mathrm{m\,s^{-2}}$. Figures marked "Schematic" are not to scale.
5. Use of calculator, mobile phone or any electronic device is not allowed. Use the space provided for rough work.

PART A : PHYSICS

25 Questions • 100 Marks

SECTION A • Multiple choice questions

This section contains TWENTY (20) questions. Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
Full marks: +4 if only the correct option is chosen; Zero marks: 0 if no option is chosen; Negative marks: −1 in all other cases.

Q.1 A ball is projected from level ground. It is level with the top edge of a vertical wall at $t = 1$ s (while rising) and again at $t = 3$ s (while falling) after projection. Neglect air resistance and take $g = 10\ \text{m s}^{-2}$. The maximum height reached by the ball **above the top edge of the wall** is

(A) 10 m (B) 20 m (C) 15 m (D) 5 m

Q.2 A block of mass 2 kg rests on the smooth inclined face of a wedge; the face makes 37° with the horizontal. The wedge is pushed along a horizontal floor with just the acceleration needed for the block to stay at rest relative to the wedge. The normal force exerted by the wedge on the block is ($\sin 37^\circ = 0.6$, $\cos 37^\circ = 0.8$, $g = 10\ \text{m s}^{-2}$)

(A) 16 N (B) 25 N

(C) 20 N (D) $\dfrac{100}{3}$ N

Q.3 A large open tank stands on level ground and its water surface is 5.0 m above the ground. Water flows out horizontally through a small hole in the side wall at a height of 1.8 m above the ground. Treat water as ideal, assume the water level stays constant and take $g = 10\ \text{m s}^{-2}$. The jet strikes the ground at a horizontal distance from the wall of

(A) 3.6 m (B) 6.0 m (C) 4.8 m (D) 2.4 m

Q.4 A solid non-conducting sphere of radius R carries a volume charge density $\rho(r) = \rho_0\left(1 - \dfrac{r}{R}\right)$, where r is the distance from its centre and $\rho_0 > 0$. Consider the magnitude of the electric field at all points inside and outside the sphere. The greatest value of the field, and the distance from the centre at which it occurs, are

(A) $\dfrac{\rho_0 R}{9\varepsilon_0}$ at $r = \dfrac{2R}{3}$ (B) $\dfrac{\rho_0 R}{12\varepsilon_0}$ at $r = R$

(C) $\dfrac{5\rho_0 R}{48\varepsilon_0}$ at $r = \dfrac{R}{2}$ (D) $\dfrac{7\rho_0 R}{64\varepsilon_0}$ at $r = \dfrac{3R}{4}$

Q.5 A battery is made by joining 10 identical cells in series, each of emf 1.5 V and internal resistance 0.5 Ω. By mistake, two of the cells are connected with reversed polarity. The battery is connected across an external resistance of 7 Ω. The current in the circuit is

(A) 1.00 A (B) 0.75 A (C) 1.25 A (D) 0.82 A

Q.6 A Zener diode of breakdown voltage 6.0 V is connected in reverse bias to a 10 V dc supply through a series resistance of 200 Ω. A load resistance of 1.0 kΩ is connected in parallel with the Zener diode. Assuming the Zener voltage stays 6.0 V in breakdown, the current through the Zener diode is

(A) 20 mA (B) 26 mA (C) 6 mA (D) 14 mA

Q.7 A proton is accelerated from rest through a potential difference of 800 V. Through what potential difference must an α-particle (mass $4m_p$, charge $2e$) be accelerated from rest so that it has the same de Broglie wavelength as the proton?

(A) 100 V (B) 400 V (C) 200 V (D) 6400 V

Q.8 A ball moving with speed v collides head-on and perfectly elastically with another ball initially at rest. In the collision the moving ball loses $\frac{8}{9}$ of its kinetic energy and **rebounds**. The ratio of the mass of the ball initially at rest to that of the moving ball is

(A) $\frac{1}{2}$ (B) 3

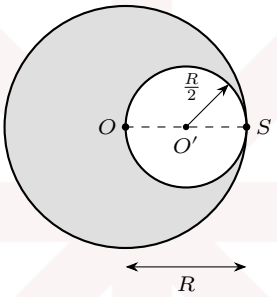
(C) 2 (D) 4

Q.9 A uniform rod of mass m and length L is hinged at one end about a smooth horizontal axis, and a small bead of mass m is fixed at its free end. The rod is held horizontal and released from rest. At the instant of release, the magnitude of the force exerted by the hinge on the rod is

(A) $\frac{5mg}{32}$ (B) $\frac{mg}{4}$

(C) $\frac{5mg}{16}$ (D) $2mg$

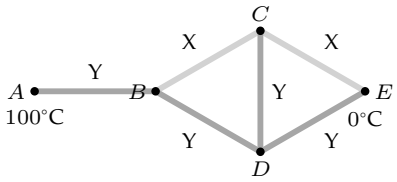
Q.10 A uniform solid sphere of density ρ and radius R, centre O, has a spherical cavity of radius $R/2$ whose centre O' is at distance $R/2$ from O. The cavity therefore passes through O and touches the outer surface at S, as shown. A small particle is released from rest just inside the cavity at S. Considering only the gravitational attraction of the sphere, the time the particle takes to reach O is



(A) $\sqrt{\frac{3}{\pi G\rho}}$ (B) $\sqrt{\frac{3}{2\pi G\rho}}$

(C) $\frac{\pi}{2}\sqrt{\frac{3}{4\pi G\rho}}$ (D) $\sqrt{\frac{6}{\pi G\rho}}$

Q.11 Six rods of identical length L and cross-sectional area A are joined as shown in the figure. Rods BC and CE are made of material X (thermal conductivity K). Rods AB, BD, DE and CD are made of material Y (thermal conductivity $3K$). End A is kept at 100°C and junction E at 0°C. The rods are insulated along their lengths and the steady state is reached. The temperature of junction B is



(A) 50°C (B) 60°C

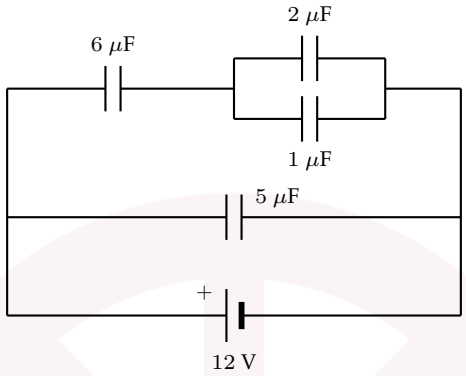
(C) $\frac{100}{3}^\circ$C (D) 40°C

Q.12 A block of mass M attached to a horizontal spring oscillates on a smooth horizontal floor with amplitude A. As the block passes through its mean position, a lump of putty of mass $3M$ is dropped vertically onto it and sticks at once. The new amplitude of oscillation is

(A) $\frac{A}{4}$ (B) $\frac{A}{\sqrt{2}}$

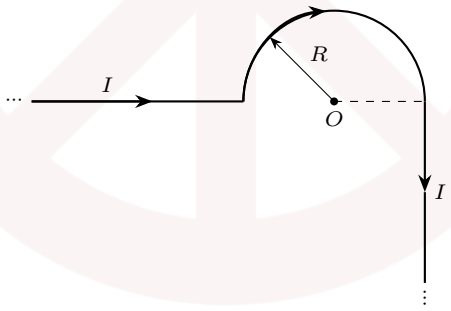
(C) A (D) $\frac{A}{2}$

Q.13 In the circuit shown, all capacitors are initially uncharged and the 12 V battery is ideal. In the steady state, the charge on the $1\ \mu$F capacitor is



(A) $16\ \mu$C (B) $8\ \mu$C (C) $24\ \mu$C (D) $4\ \mu$C

Q.14 A very long wire carrying current I is bent into the shape shown and lies in the plane of the page. A straight part comes in along a line passing through O and stops at distance R from O; it continues as a semicircle of radius R centred at O; it then leaves along a straight line perpendicular to the first part, tangent to the semicircle at its other end, and goes to infinity. The magnitude of the magnetic field at O is



(A) $\frac{\mu_0 I}{4R}$ (B) $\frac{\mu_0 I}{4\pi R}(\pi+2)$

(C) $\frac{\mu_0 I}{4\pi R}(\pi+1)$ (D) $\frac{\mu_0 I}{4\pi R}(\pi-1)$

Q.15 A bulb rated 100 V, 50 W has to be operated at its rated values on a 200 V (rms), 50 Hz ac supply. A pure inductor is connected in series with it for this purpose. Treating the bulb as a pure resistor, the inductance needed is

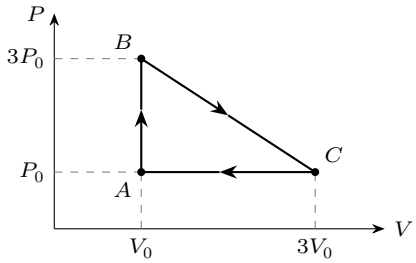
(A) $\frac{2\sqrt{3}}{\pi}$ H (B) $\frac{4}{\pi}$ H

(C) $\frac{\sqrt{3}}{\pi}$ H (D) $\frac{2}{\pi}$ H

Q.16 In the u–v method for a convex lens, a student measures the object distance as (30.0 ± 0.1) cm and the image distance as (60.0 ± 0.2) cm (both magnitudes; the image is real). The focal length is found from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ (magnitudes). The maximum possible absolute error in f, found by differentiating this relation, is closest to

(A) 0.30 cm (B) 0.10 cm (C) 0.13 cm (D) 0.067 cm

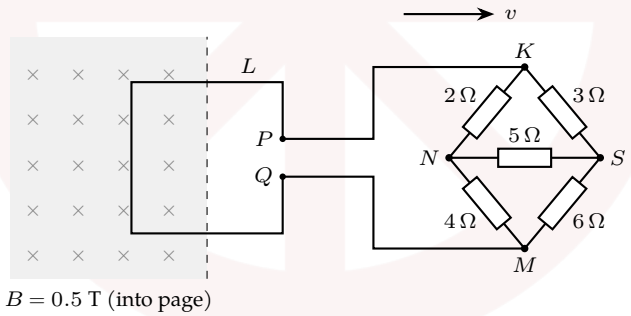
Q.17 One mole of a monatomic ideal gas is taken through the cycle $A \to B \to C \to A$ shown on the P–V diagram: $A(V_0, P_0)$, $B(V_0, 3P_0)$ and $C(3V_0, P_0)$, where BC is a straight line. The efficiency of the cycle is



(A) $\frac{2}{7}$ (B) $\frac{2}{3}$

(C) $\frac{4}{15}$ (D) $\frac{1}{5}$

Q.18 A square conducting loop L of side 10 cm has a small gap PQ in its right side; its wire has a total resistance of $1.4\ \Omega$. Terminals P and Q are joined by resistanceless wires to nodes K and M of the resistor network shown. The whole arrangement moves to the right with a constant speed $v = 5\ \text{cm s}^{-1}$. At the instant shown, the left side of L is inside a uniform magnetic field of 0.5 T directed into the page, while the right side of L, the connecting wires and the network are outside the field. Neglect self-inductance. The current in the loop at this instant is



(A) 250 μA (B) 694 μA (C) 1786 μA (D) 500 μA

Q.19 A potentiometer wire AB is 400 cm long and has a resistance of $8\ \Omega$. It is connected in series with a $4\ \Omega$ resistor and a driver cell of emf 2.4 V (negligible internal resistance). A cell of emf E and internal resistance r is balanced at 250 cm from A when it is on open circuit. When a $6\ \Omega$ resistor is connected across this cell, the balance point shifts to 200 cm. Then (E, r) is

(A) $(1.0\ \text{V},\ 1.5\ \Omega)$ (B) $(1.0\ \text{V},\ 1.2\ \Omega)$ (C) $(1.5\ \text{V},\ 1.5\ \Omega)$ (D) $(1.5\ \text{V},\ 1.2\ \Omega)$

Q.20 In a Young's double-slit experiment in air, a thin glass sheet of refractive index 1.5 placed over one slit shifts the central bright fringe to the position previously occupied by the 5^{th} bright fringe. The whole apparatus, with the sheet still in place, is now immersed in water (refractive index $4/3$). In water, the central bright fringe is displaced from the centre of the screen by how many fringe widths? (Fringe width measured in water.)

(A) 5 (B) $\frac{5}{3}$

(C) $\frac{5}{4}$ (D) $\frac{20}{3}$

SECTION B • Numerical value questions

This section contains FIVE (05) questions. The answer to each question is an INTEGER (it may be negative). All five are compulsory.
Full marks: +4 if only the correct value is entered; Zero marks: 0 if unattempted; Negative marks: −1 in all other cases.

Q.21 A cylinder with a frictionless, movable piston contains a mixture of 4 g of helium and 28 g of nitrogen. Treat both as ideal gases, with nitrogen rigid diatomic (no vibration). First, 1000 J of heat is supplied slowly with the piston free, so the pressure stays constant. The piston is then clamped and a further 1000 J of heat is supplied. There is no heat loss. Take $R = \dfrac{25}{3}$ J mol^{-1} K^{-1}. Find the total rise in temperature of the mixture, in kelvin.

Answer: _____

Q.22 A uniform solid sphere of radius 10 cm rests on a rough horizontal floor. It is given a sharp horizontal impulse whose line of action lies in the vertical plane through the centre, at height h above the floor. Find h (in cm) for which the sphere starts rolling without slipping immediately, so that friction plays no role.

Answer: _____

Q.23 A point object is placed on the principal axis of a thin convex lens of focal length 12 cm, at a distance of 18 cm from it. A convex mirror is placed coaxially on the other side of the lens, 16 cm from the lens. The final image formed by the system coincides with the object itself. Find the magnitude of the focal length of the convex mirror, in cm.

Answer: _____

Q.24 A sample initially contains equal numbers of nuclei of two radioactive species A (half-life 10 min) and B (half-life 20 min); neither decays into the other. After how many minutes will the activity of A become one-fourth of the activity of B?

Answer: _____

Q.25 A car moves at 30 m s^{-1} straight towards a large vertical wall, sounding a horn of frequency 600 Hz. A cyclist behind the car, on the same straight line, rides towards the wall at 6 m s^{-1} (the car is between the cyclist and the wall). The cyclist hears the sound coming directly from the car and the sound reflected from the wall. Speed of sound in still air $= 330$ m s^{-1}; there is no wind. Find the beat frequency (in Hz) heard by the cyclist.

Answer: _____

PART B : CHEMISTRY

25 Questions • 100 Marks

SECTION A • Multiple choice questions

This section contains TWENTY (20) questions. Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
Full marks: +4 if only the correct option is chosen; Zero marks: 0 if no option is chosen; Negative marks: −1 in all other cases.

Q.26 Which one of the following alkenes does **not** show geometrical (cis–trans) isomerism?
(A) $BrCH{=}CHCl$
(B) $CH_3CH{=}C(CH_3)COOH$
(C) $CH_3CH{=}C(CH_2CH_3)_2$
(D) $CH_3CH{=}C(CH_3)CH_2CH_3$

Q.27 In which option are **all three** orders correct? (Size means atomic radius for atoms and ionic radius for ions.)
(A) First ionisation enthalpy: B < C < O < N; size: Ga < Al < In; magnitude of electron gain enthalpy: O < S < Cl
(B) First ionisation enthalpy: Na < Al < Mg; size: Ga < Al < In; magnitude of electron gain enthalpy: Br < Cl < F
(C) First ionisation enthalpy: Be < B < C; size: Ga < Al < In; magnitude of electron gain enthalpy: O < S < Cl
(D) First ionisation enthalpy: B < C < O < N; size: $Na^+ < Mg^{2+} < Al^{3+}$; magnitude of electron gain enthalpy: O < S < Cl

Q.28 The correct decreasing order of the H–X–H bond angle in H_2O, NH_3, PH_3 and H_2S is
(A) $NH_3 > H_2O > H_2S > PH_3$
(B) $H_2O > NH_3 > H_2S > PH_3$
(C) $NH_3 > PH_3 > H_2O > H_2S$
(D) $NH_3 > H_2O > PH_3 > H_2S$

Q.29 Benzene is treated with 1-chloropropane, $CH_3CH_2CH_2Cl$, in the presence of anhydrous $AlCl_3$. The major product is
(A) n-Propylbenzene, $C_6H_5CH_2CH_2CH_3$
(B) Isopropylbenzene (cumene), $C_6H_5CH(CH_3)_2$
(C) Ethylbenzene, $C_6H_5CH_2CH_3$
(D) tert-Butylbenzene, $C_6H_5C(CH_3)_3$

Q.30 Which of the following statements is **incorrect**?
(A) In neutral or faintly alkaline solution, $KMnO_4$ oxidises iodide ions to iodine.
(B) Acidified $K_2Cr_2O_7$ oxidises iodide ions to iodine.
(C) Acidified $KMnO_4$ oxidises oxalate ions to CO_2 at about 333 K.
(D) On acidification, an aqueous chromate solution changes to dichromate: $2\,CrO_4^{2-} + 2\,H^+ \longrightarrow Cr_2O_7^{2-} + H_2O$.

Q.31 Given below are two statements.
Statement I: When a globular protein is denatured, for example by heating, its primary structure is destroyed.
Statement II: Amylose, which makes up about 15–20% of starch, is the water-soluble component of starch.
Choose the correct option.
(A) Both Statement I and Statement II are false
(B) Both Statement I and Statement II are true
(C) Statement I is false but Statement II is true
(D) Statement I is true but Statement II is false

Q.32 2,2-Dimethylpentan-3-ol, $(CH_3)_3C{-}CH(OH){-}CH_2CH_3$, is subjected to two separate reactions.
P: heating with concentrated HBr.
Q: heating with concentrated H_2SO_4.
The major organic products of P and Q, respectively, are
(A) 3-Bromo-2,2-dimethylpentane and 4,4-dimethylpent-2-ene
(B) 2-Bromo-2,3-dimethylpentane and 2,3-dimethylpent-2-ene
(C) 2-Bromo-2,3-dimethylpentane and 2,3-dimethylpent-1-ene
(D) 3-Bromo-2,2-dimethylpentane and 2,3-dimethylpent-2-ene

Q.33 An amine X, C_3H_9N, reacts with benzenesulphonyl chloride to give a product that is **insoluble** in aqueous KOH. X is
(A) Propan-1-amine
(B) N,N-Dimethylmethanamine
(C) Propan-2-amine
(D) N-Methylethanamine

Q.34 In a hydrogen-like ion, the photon emitted in the transition $n = 3 \rightarrow n = 2$ has energy 68.0 eV. The minimum energy needed to remove the electron from the $n = 2$ level of this ion is (take the ground-state energy of H as -13.6 eV)
(A) 30.6 eV (B) 489.6 eV (C) 54.4 eV (D) 122.4 eV

Q.35 What mass of iron(II) oxalate, FeC_2O_4, is completely oxidised by 50.0 mL of 0.100 M $K_2Cr_2O_7$ in acidic medium? (Fe = 56, C = 12, O = 16; the products are Fe^{3+}, CO_2 and Cr^{3+}.)
(A) 1.44 g (B) 2.16 g (C) 0.72 g (D) 4.32 g

Q.36 For $N_2O_4(g) \rightleftharpoons 2\,NO_2(g)$ at a certain temperature, the equilibrium mixture at a total pressure of 2.0 atm has a vapour density of 34.5 (relative to H_2). The value of K_p at this temperature is (N = 14, O = 16)
(A) 0.50 atm (B) 2.0 atm (C) 1.0 atm (D) 0.25 atm

Q.37 For the gas-phase reaction $2\,A(g) + B(g) \longrightarrow 2\,C(g)$ at 300 K, $\Delta U^\circ = -10.5$ kJ mol^{-1} and $\Delta S^\circ = -44.1$ J K^{-1} mol^{-1}. Treat all gases as ideal and take $R = 8.314$ J K^{-1} mol^{-1}. Under standard conditions at 300 K, ΔG° and the nature of the reaction are
(A) $+2.73$ kJ mol^{-1}; non-spontaneous
(B) $+0.24$ kJ mol^{-1}; non-spontaneous
(C) -26.2 kJ mol^{-1}; spontaneous
(D) $+5.22$ kJ mol^{-1}; non-spontaneous

Q.38 The correct **increasing** order of the wavelength of light absorbed by the complexes $[Co(CN)_6]^{3-}$, $[Co(NH_3)_6]^{3+}$, $[Co(H_2O)_6]^{3+}$ and $[CoF_6]^{3-}$ is
(A) $[CoF_6]^{3-} < [Co(H_2O)_6]^{3+} < [Co(NH_3)_6]^{3+} < [Co(CN)_6]^{3-}$
(B) $[Co(CN)_6]^{3-} < [Co(NH_3)_6]^{3+} < [Co(H_2O)_6]^{3+} < [CoF_6]^{3-}$
(C) $[Co(NH_3)_6]^{3+} < [Co(CN)_6]^{3-} < [Co(H_2O)_6]^{3+} < [CoF_6]^{3-}$
(D) $[Co(CN)_6]^{3-} < [Co(H_2O)_6]^{3+} < [Co(NH_3)_6]^{3+} < [CoF_6]^{3-}$

Q.39 Consider the following statements.
I. The thermal stability of the hydrogen halides decreases in the order HF > HCl > HBr > HI.
II. The bond dissociation enthalpy of F_2 is greater than that of Cl_2.
III. H_3PO_3 is dibasic and acts as a reducing agent.
IV. Among the group 16 hydrides, H_2O has the highest boiling point but the lowest acidic character.
V. SF_6 is readily hydrolysed by water.
The correct statements are
(A) I, II and III only
(B) II, IV and V only
(C) III, IV and V only
(D) I, III and IV only

Q.40 A compound X, $C_5H_{10}O$, gives an orange precipitate with 2,4-DNP reagent and a yellow precipitate with I_2/NaOH, but does not reduce Tollens' reagent. Wolff–Kishner reduction of X gives 2-methylbutane. X is
(A) 3-Methylbutan-2-one
(B) Pentan-2-one
(C) Pentan-3-one
(D) 3-Methylbutanal

Q.41 A solid salt X shows the following behaviour.
(i) With dilute H_2SO_4 it gives brisk effervescence. The gas is colourless and odourless, turns lime water milky, and does **not** turn acidified potassium dichromate paper green.
(ii) A solution of X in dilute HCl gives no precipitate when H_2S is passed through it.
(iii) After NH_4Cl and excess NH_4OH are added to this solution, no precipitate forms. When H_2S is then passed through the ammoniacal solution, a **white** precipitate appears.
X is
(A) $ZnSO_3$ (B) $MnCO_3$ (C) $ZnCO_3$ (D) $BaCO_3$

Q.42 Consider the following statements about the 3d transition series.
I. Manganese shows the largest number of oxidation states, from +2 to +7.
II. Cu^{+} is stable in aqueous solution because it has a completely filled $3d^{10}$ configuration.
III. $E^{\circ}(Mn^{3+}/Mn^{2+})$ is much more positive than $E^{\circ}(Fe^{3+}/Fe^{2+})$, because of the extra stability of the d^5 ion Mn^{2+}.
IV. Zinc has the highest enthalpy of atomisation in the series, because its d orbitals are completely filled.
The correct statements are
(A) I and III only
(B) I, II and III only
(C) III and IV only
(D) I, III and IV only

Q.43 For the reaction A $\rightarrow$ products at 300 K, the half-life is 200 s when $[A]_0 = 0.10$ M and 400 s when $[A]_0 = 0.050$ M. At 320 K, starting with $[A]_0 = 0.20$ M, the concentration of A falls to 0.050 M in 75 s. The activation energy of the reaction is closest to ($R = 8.314\ J\,K^{-1}\,mol^{-1}$, $\ln 2 = 0.693$)
(A) 27.7 kJ mol^{-1}
(B) 55.3 kJ mol^{-1}
(C) 110.6 kJ mol^{-1}
(D) 66.8 kJ mol^{-1}

Q.44 Consider the following sequence.
Step 1: Toluene $\xrightarrow{\text{(i) } KMnO_4/KOH,\ \Delta;\ \text{(ii) } H_3O^+}$ A
Step 2: A $\xrightarrow{\text{conc. } HNO_3/\text{conc. } H_2SO_4,\ \Delta}$ B
Step 3: B $\xrightarrow{\text{(i) Sn/HCl; (ii) neutralise}}$ C
Step 4: C $\xrightarrow{NaNO_2/HCl,\ 0\text{–}5\ ^\circ C}$ D $\xrightarrow{H_2O,\ \text{warm}}$ E
The major product E is
(A) 2-Hydroxybenzoic acid
(B) 3-Chlorobenzoic acid
(C) 4-Hydroxybenzoic acid
(D) 3-Hydroxybenzoic acid

Q.45 At 298 K the limiting molar conductivities of HCl, NaCl and CH_3COONa are 425.9, 126.4 and 91.0 S cm^2 mol^{-1} respectively. A 0.010 M solution of acetic acid has conductivity 1.60×10^{-4} S cm^{-1} at 298 K. The pH of this acetic acid solution is closest to ($\log 4.1 = 0.613$)
(A) 2.39
(B) 4.39
(C) 3.39
(D) 3.60

SECTION B • Numerical value questions

This section contains FIVE (05) questions. The answer to each question is an INTEGER (it may be negative). All five are compulsory.
Full marks: +4 if only the correct value is entered; Zero marks: 0 if unattempted; Negative marks: −1 in all other cases.

Q.46 Find the total number of stereoisomers (geometrical and optical counted together) summed over the following four complexes: $[Cr(C_2O_4)_2Cl_2]^{3-}$, $[Co(NH_3)_3Cl_3]$, $[Co(en)_3]^{3+}$ and $[Co(NH_3)_4Cl_2]^{+}$. ($C_2O_4^{2-}$ and en are symmetrical bidentate ligands.)
Answer: _____

Q.47 How many of the following species have a **square planar** geometry? XeF_4, $[Ni(CN)_4]^{2-}$, SF_4, $[PtCl_4]^{2-}$, BrF_4^{-}, ICl_4^{-}, $[NiCl_4]^{2-}$, CCl_4, XeO_4, $[Zn(NH_3)_4]^{2+}$
Answer: _____

Q.48 At 298 K, 50.0 mL of 0.20 M acetic acid is mixed with 50.0 mL of 0.20 M NaOH. Given $K_a(CH_3COOH) = 1.0 \times 10^{-5}$ and $K_w = 1.0 \times 10^{-14}$, find the pH of the resulting solution.
Answer: _____

Q.49 Benzoic acid dimerises in benzene. When 1.22 g of benzoic acid is dissolved in 40.0 g of benzene, the freezing point is lowered by 0.768 K. Given K_f(benzene) = 5.12 K kg mol^{-1} and molar mass of benzoic acid = 122 g mol^{-1}, find the percentage of benzoic acid molecules that are associated as dimers.
Answer: _____

Q.50 Consider the following sequence.

Step 1: $CH_3CHO \xrightarrow{\text{(i) dil. NaOH; (ii) heat}} A \xrightarrow{H_2\ (\text{excess})/Ni} B$

Step 2: $B \xrightarrow{PCC} C \xrightarrow{\text{(i) } CH_3MgBr;\ \text{(ii) } H_3O^+} D$

Step 3: $D \xrightarrow{\text{conc. } H_2SO_4,\ 443\ K} E\ (\text{major}) \xrightarrow{\text{(i) } O_3;\ \text{(ii) } Zn/H_2O} F + G$

F and G are different carbonyl compounds. Find the molar mass (in $g\,mol^{-1}$) of the **heavier** of F and G. (C = 12, H = 1, O = 16)

Answer: _____

PART C : MATHEMATICS

25 Questions • 100 Marks

SECTION A • Multiple choice questions

This section contains TWENTY (20) questions. Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
Full marks: +4 if only the correct option is chosen; Zero marks: 0 if no option is chosen; Negative marks: −1 in all other cases.

Q.51 Let $A = \{1, 2, 3, 4, 5, 6\}$ and let R be the smallest equivalence relation on A that contains the ordered pairs $(1, 2)$, $(2, 3)$ and $(4, 5)$. The number of ordered pairs in R is

(A) 14 (B) 13 (C) 11 (D) 9

Q.52 Let z be a complex number such that $|z| + iz = 1 + 3i$. Then z is equal to

(A) $3 - 4i$ (B) $4 + 3i$ (C) $3 + 4i$ (D) $-3 + 4i$

Q.53 The mean and the variance of 10 observations are 12 and 16 respectively. Later it was found that one observation had been wrongly recorded as 18 instead of the correct value 8. The correct variance is

(A) 16 (B) 13 (C) 12 (D) 15

Q.54 Q is the image of the point $P(4, 1)$ in the line $x+2y = 1$. The perpendicular distance of Q from the line $3x-4y = 3$ is

(A) 1 (B) $\frac{13}{5}$

(C) $\frac{9}{5}$ (D) 3

Q.55 The coefficient of x^3 in the expansion of $(1 + x^3)\left(x^2 - \frac{2}{x}\right)^6$ is

(A) -160 (B) 240 (C) 400 (D) 80

Q.56 The distance of the point $P(4, -1, 5)$ from the plane $x - y + z = 5$, measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$, is

(A) $\frac{5}{\sqrt{3}}$ (B) 5

(C) 7 (D) $\frac{5}{7}$

Q.57 For a sequence $\{a_n\}$, the sum of the first n terms is $S_n = n^2 + 3n$ for every $n \in \mathbb{N}$. Then $\sum_{k=1}^{20} \frac{1}{a_k\, a_{k+1}}$ is equal to

(A) $\frac{5}{44}$ (B) $\frac{1}{11}$

(C) $\frac{21}{88}$ (D) $\frac{5}{22}$

Q.58 Two distinct numbers are chosen at random from $\{1, 2, 3, \ldots, 20\}$. Given that their sum is even, the probability that both chosen numbers are prime is

(A) $\frac{21}{190}$ (B) $\frac{14}{45}$

(C) $\frac{7}{30}$ (D) $\frac{7}{19}$

Q.59 The set of all real values of k for which both roots of $x^2-2kx+k^2+k-5=0$ are real and lie in the open interval $(-1,5)$ is
(A) $(1,5]$ (B) $(1,4)$ (C) $(-4,4)$ (D) $(1,5)$

Q.60 Let $F(x)=\int e^x \dfrac{x^2+3x+3}{(x+2)^2}\,dx$ for $x>-2$, with $F(0)=\dfrac{1}{2}$. Then $F(1)$ is equal to
(A) $\dfrac{2e}{3}$ (B) $\dfrac{3e}{4}$
(C) $\dfrac{2e}{3}-\dfrac{1}{2}$ (D) $\dfrac{e}{3}$

Q.61 Considering principal values, the set of all real x satisfying $2\tan^{-1}x+\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)=\pi$ is
(A) $\{1\}$ (B) $(-\infty,-1]\cup[1,\infty)$ (C) $(1,\infty)$ (D) $[1,\infty)$

Q.62 A point P on the line $x+y=6$ is such that the two tangents drawn from P to the circle $x^2+y^2-2x-2y-2=0$ are perpendicular to each other. The equation of the circle passing through P and the two points of contact of these tangents is
(A) $x^2+y^2-4x-4y+4=0$ (B) $x^2+y^2-2x-2y-6=0$
(C) $x^2+y^2-4x-4y+6=0$ (D) $x^2+y^2-6x-6y+16=0$

Q.63 Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{k}$. A vector $\vec{c}$ satisfies $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=4$. If θ is the angle between $\vec{a}$ and $\vec{c}$, then
(A) $\sin\theta=\dfrac{2\sqrt{2}}{3}$ (B) $\cos\theta=\dfrac{1}{3}$
(C) $\sin\theta=\dfrac{1}{3}$ (D) $\tan\theta=2\sqrt{2}$

Q.64 Let $a=\lim\limits_{x\to0}x^2\left[\dfrac{3}{x^2}\right]$, where $[t]$ denotes the greatest integer less than or equal to t, and let

$$b=\lim_{x\to0}\frac{e^{2x^3}-\left(1-3x^3\right)^{1/3}+\left(\sqrt{1-x^2}-1\right)\sin 2x}{x\sin^2 x}.$$

Then $a+b$ is equal to
(A) 5 (B) 4 (C) 6 (D) 3

Q.65 Let $A=\begin{bmatrix}4 & 1\\ 5 & 2\end{bmatrix}$ and, for $p\in\mathbb{N}$, let $B_p=A^p\,\underbrace{\text{adj}(\text{adj}(\cdots\text{adj}}_{p\text{ times}}(A)\cdots))$. Which of the following statements is **incorrect**?
(A) $\text{tr}(B_1)=6$ (B) $\text{adj}(\text{adj}(B_2))=A^3$
(C) $\det(B_2)=27$ (D) $\displaystyle\sum_{p=1}^{\infty}\det\left(B_p^{-1}\right)=\frac{1}{3}$

Q.66 A hyperbola has its centre at the origin, its transverse axis along the x-axis and its foci at $(\pm5,0)$. It passes through the point $P\left(4\sqrt{2},\,3\right)$. The tangent to the hyperbola at P meets the x-axis at T and the y-axis at U. If O is the origin, the area of triangle OTU is
(A) $6\sqrt{2}$ (B) $3\sqrt{2}$
(C) $\dfrac{3\sqrt{2}}{2}$ (D) 12

Q.67 Let $f(x) = \max\{x^2 - 2,\ |x|\}$ and $g(x) = x\,f(x)$ for $x \in (-3, 3)$. If m is the number of points in $(-3, 3)$ at which f is not differentiable and n is the number of points in $(-3, 3)$ at which g is not differentiable, then $m + n$ is equal to
(A) 4 (B) 3 (C) 6 (D) 5

Q.68 Let $y = y(x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, be the solution of $\frac{dy}{dx} + 2y\tan x = \sin x$ with $y\left(\frac{\pi}{4}\right) = 0$. Then the maximum value of $y(x)$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is
(A) $\frac{\sqrt{2}}{4}$ (B) $\frac{1}{8}$
(C) $\frac{\sqrt{2}}{8}$ (D) 0

Q.69 Let $f(x) = \int_0^x (t^2 - 5t + 6)\,e^t\,dt$, $x \in \mathbb{R}$. If M and m are respectively the local maximum value and the local minimum value of f, then $M - m$ is equal to
(A) $e^2(3 - e)$ (B) $e^2(e - 2)$ (C) $e^3 - e^2$ (D) $3e^2 - 2e^3$

Q.70 Let $f(x) = 2\sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x)\,f(t)\,dt$ for all real x. Match List-I with List-II.

List-I	List-II
(P) $\alpha + \beta$, where $[\alpha, \beta]$ is the range of f	(1) 0
(Q) $f(3\pi/2)$	(2) 2/3
(R) $f(\pi)$	(3) 4/3
(S) $f(5\pi/4)$	(4) $\sqrt{2}$
	(5) $-2/3$

Choose the correct option.
(A) (P)–1, (Q)–5, (R)–3, (S)–4 (B) (P)–1, (Q)–2, (R)–3, (S)–4
(C) (P)–2, (Q)–1, (R)–3, (S)–4 (D) (P)–4, (Q)–2, (R)–3, (S)–1

SECTION B • Numerical value questions

This section contains FIVE (05) questions. The answer to each question is an INTEGER (it may be negative). All five are compulsory.
Full marks: +4 if only the correct value is entered; Zero marks: 0 if unattempted; Negative marks: −1 in all other cases.

Q.71 The number of eight-digit numbers that can be formed using each of the digits $1, 2, 3, 4, 5, 6, 7, 8$ exactly once, and that are divisible by 275, is

Answer: _____

Q.72 Consider the lines $L_1 :\ \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k})$ and $L_2 :\ \vec{r} = (2\hat{i} + \hat{k}) + \mu(2\hat{i} + \hat{j} + \hat{k})$. Let d be the shortest distance between L_1 and L_2. Let Π be the plane containing L_1 and parallel to L_2, and let B be the image of the point $A(1, 1, 1)$ in Π. Then the value of $3d^2 + AB^2$ is

Answer: _____

Q.73 The parabola $y^2 = 8x$ and the ellipse $x^2 + 4y^2 = 32$ have exactly two common tangents. The four points at which these common tangents touch the two curves are the vertices of a quadrilateral. The area of this quadrilateral (in square units) is

Answer: _____

Q.74 Let $f : (0, \infty) \to \mathbb{R}$ be a differentiable function satisfying $f(xy) = f(x) + f(y)$ for all $x, y > 0$, with $f'(1) = 2$. Let $g(x) = x^3 - 7x^2 + 14x - 8$. If $\mathcal{A}$ is the area of the region $\{(x, y) :\ 0 < x \le 1,\ f(x) \le y \le |g(x)|\}$, then $12\mathcal{A}$ is equal to

Answer: _____

Q.75 Let $f : \mathbb{R} \to \mathbb{R}$ be an odd function with $f(1) = 1$ and $f\left(\frac{x}{x-1}\right) = x\, f(x)$ for every $x < 0$. Let

$$S = f(1)f\left(\frac{1}{60}\right) + f\left(\frac{1}{2}\right) f\left(\frac{1}{59}\right) + f\left(\frac{1}{3}\right) f\left(\frac{1}{58}\right) + \cdots + f\left(\frac{1}{30}\right) f\left(\frac{1}{31}\right).$$

If $S = \frac{2^K}{M!}$ for positive integers K and M, with M as small as possible, then $K + M$ is equal to

Answer: _____

— END OF PAPER —