॥आचार्य सारथी॥
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JEE Main Free Mock 2

Answer Key & Detailed Solutions · Free mock test

Answer key

Physics				
1 : D	**2** : B	**3** : C	**4** : A	**5** : B
6 : D	**7** : A	**8** : C	**9** : C	**10** : A
11 : B	**12** : D	**13** : B	**14** : C	**15** : A
16 : D	**17** : C	**18** : D	**19** : A	**20** : B
21 : 50	**22** : 14	**23** : 10	**24** : 60	**25** : 112

Chemistry				
26 : C	**27** : A	**28** : D	**29** : B	**30** : A
31 : C	**32** : B	**33** : D	**34** : D	**35** : A
36 : C	**37** : B	**38** : B	**39** : D	**40** : A
41 : C	**42** : A	**43** : B	**44** : D	**45** : C
46 : 9	**47** : 5	**48** : 9	**49** : 80	**50** : 58

Mathematics				
51 : A	**52** : C	**53** : B	**54** : D	**55** : D
56 : B	**57** : A	**58** : C	**59** : B	**60** : A
61 : D	**62** : C	**63** : C	**64** : A	**65** : D
66 : B	**67** : D	**68** : C	**69** : A	**70** : B
71 : 180	**72** : 37	**73** : 120	**74** : 61	**75** : 117

Q.1–20, 26–45, 51–70: MCQ (+4/−1) • Q.21–25, 46–50, 71–75: integer answers (+4/−1).

Chapter, difficulty and time map

Q	Code	Chapter / concept	Level	Time (s)
1	P01	Kinematics – symmetry of projectile motion	Foundation	60
2	P02	Laws of motion – pseudo force on an accelerated wedge	Foundation	75
3	P03	Fluids – Torricelli efflux and range	Foundation	90
4	P04	Electrostatics – Gauss's law for a non-uniform charge distribution	Challenging	150
5	P05	Current electricity – cells in series with reversed cells	Foundation	75
6	P06	Semiconductors – Zener diode as voltage regulator	Foundation	75
7	P07	Dual nature – de Broglie wavelength of accelerated particles	Foundation	75

8	P08	Work, energy and power – elastic head-on collision	Standard	105
9	P09	Rotational motion – hinge reaction at release	Standard	150
10	P10	Gravitation – field inside a spherical cavity (superposition)	Standard	120
11	P11	Heat transfer – thermal resistance network (balanced bridge)	Standard	120
12	P12	Simple harmonic motion – mass added at mean position	Standard	105
13	P13	Capacitance – series-parallel combination	Standard	90
14	P14	Magnetic effects of current – Biot–Savart for a bent wire	Standard	120
15	P15	Alternating current – choke coil in series with a bulb	Standard	105
16	P16	Experimental skills – error in focal length (u–v method)	Standard	120
17	P17	Thermodynamics – efficiency of a cycle with a straight-line process	Challenging	210
18	P18	Electromagnetic induction – motional emf driving a bridge network	Challenging	180
19	P19	Current electricity – potentiometer: emf and internal resistance	Challenging	180
20	P20	Wave optics – YDSE with a thin sheet, apparatus immersed in water	Challenging	180
21	P21	Thermodynamics – heat capacities of a gas mixture (isobaric then isochoric)	Standard	150
22	P22	Rotational motion – impulse giving immediate pure rolling	Standard	120
23	P23	Ray optics – lens–convex mirror combination (image on object)	Standard	120
24	P24	Nuclei – activity of two radioactive species	Standard	120
25	P25	Waves – Doppler effect with reflection and beats	Challenging	180
26	C01	Isomerism – geometrical isomerism in alkenes	Foundation	45
27	C02	Periodicity – trends in ionisation enthalpy, radius and electron gain enthalpy (with exceptions)	Standard	90
28	C03	Chemical bonding – VSEPR and bond angles of hydrides	Foundation	50
29	C04	Hydrocarbons – Friedel–Crafts alkylation and rearrangement	Foundation	45
30	C05	d-Block – oxidising action of KMnO4 and K2Cr2O7	Foundation	50
31	C06	Biomolecules – protein denaturation and starch	Foundation	45
32	C07	Alcohols – carbocation rearrangement in substitution and dehydration	Challenging	120
33	C08	Amines – Hinsberg test	Foundation	60
34	C09	Atomic structure – Bohr model for hydrogen-like ions	Standard	100
35	C10	Redox – n-factor and equivalents (dichromate titration)	Standard	100
36	C11	Chemical equilibrium – degree of dissociation from vapour density	Standard	110
37	C12	Thermodynamics – ΔH from ΔU and Gibbs energy	Standard	120
38	C13	Coordination compounds – spectrochemical series and colour	Standard	80
39	C14	p-Block – groups 15, 16 and 17 (NCERT statements)	Standard	110
40	C15	Aldehydes and ketones – structure from tests	Standard	100
41	C16	Practical chemistry – qualitative salt analysis	Standard	110
42	C17	d-Block – oxidation states, electrode potentials and atomisation enthalpy (NCERT statements)	Standard	100
43	C18	Chemical kinetics – order from half-life and Arrhenius equation	Challenging	200
44	C19	Multi-step organic conversion – aromatic substitution and diazonium chemistry	Challenging	150
45	C20	Electrochemistry – Kohlrausch's law, degree of dissociation and pH	Challenging	200
46	C21	Coordination compounds – geometrical and optical isomer count	Standard	90
47	C22	Chemical bonding – VSEPR and square planar species	Foundation	75
48	C23	Ionic equilibrium – neutralisation and salt hydrolysis	Standard	100
49	C24	Solutions – association and depression in freezing point	Standard	120
50	C25	Multi-step organic conversion – aldol, Grignard, dehydration, ozonolysis	Challenging	180
51	M01	Sets and relations – equivalence closure	Foundation	75
52	M02	Complex numbers – modulus equation	Foundation	80
53	M03	Statistics – corrected mean and variance	Foundation	90
54	M04	Straight lines – image of a point and distance	Foundation	90
55	M05	Binomial theorem – coefficient in a product	Foundation	100
56	M06	3D geometry – distance measured along a line	Foundation	100
57	M07	Sequences and series – nth term from S(n), telescoping sum	Standard	110
58	M08	Probability – conditional probability with counting	Standard	110
59	M09	Quadratic equations – location of roots	Standard	140
60	M10	Indefinite integration – exponential times (g + g′) form	Standard	120
61	M11	Inverse trigonometric functions – principal value branches	Standard	120
62	M12	Circles – director circle, points of contact and the circle on CP as diameter	Standard	120
63	M13	Vector algebra – vector equations (cross and dot products given)	Standard	150
64	M14	Limits – GIF sandwich and series expansions	Standard	140
65	M15	Matrices and determinants – adjoint chains	Standard	150
66	M16	Hyperbola – equation from foci and a point; tangent	Standard	150
67	M17	Continuity and differentiability – max function and product	Standard	170
68	M18	Differential equations – linear DE, then optimisation	Challenging	180
69	M19	Application of derivatives – extrema of an integral function	Challenging	180
70	M20	Definite integration – integral equation, odd/even parts (List match)	Challenging	200
71	M21	Permutations – divisibility by 275 (rules for 25 and 11)	Standard	240
72	M22	3D geometry – plane through a line parallel to another; image; shortest distance	Standard	210
73	M23	Conics – common tangents of a parabola and an ellipse	Challenging	240
74	M24	Area under curves – functional equation and an improper region	Challenging	240
75	M25	Functions – functional equation with a binomial sum	Challenging	240

PART A : PHYSICS

Detailed solutions

Q.1

Kinematics – symmetry of projectile motion • Foundation • 60 s **Answer: D**

A ball is projected from level ground. It is level with the top edge of a vertical wall at $t = 1$ s (while rising) and again at $t = 3$ s (while falling) after projection. Neglect air resistance and take $g = 10\ \text{m s}^{-2}$. The maximum height reached by the ball **above the top edge of the wall** is

(A) 10 m (B) 20 m (C) 15 m (D) 5 m

Solution

The vertical motion is symmetric about the top of the path, so the ball is highest at

$$t_{\text{top}} = \frac{1+3}{2} = 2\ \text{s}.$$

From the top ($t = 2$ s) it falls freely for 1 s to reach the wall's height again ($t = 3$ s):

$$\Delta h = \tfrac{1}{2} g (1)^2 = 5\ \text{m}.$$

(Check: wall height $= \tfrac{1}{2} g t_1 t_2 = 15$ m, maximum height $= \tfrac{1}{2} g (2)^2 = 20$ m, difference 5 m.)

Answer: (D)

Common mistake: Giving the maximum height above the ground (20 m) or the wall height (15 m) instead of the height above the wall.

Q.2

Laws of motion – pseudo force on an accelerated wedge • Foundation • 75 s **Answer: B**

A block of mass 2 kg rests on the smooth inclined face of a wedge; the face makes 37° with the horizontal. The wedge is pushed along a horizontal floor with just the acceleration needed for the block to stay at rest relative to the wedge. The normal force exerted by the wedge on the block is ($\sin 37^\circ = 0.6$, $\cos 37^\circ = 0.8$, $g = 10\ \text{m s}^{-2}$)

(A) 16 N (B) 25 N

(C) 20 N (D) $\frac{100}{3}$ N

Solution

The incline is smooth, so only gravity and the normal force N act on the block. The block moves with the wedge, so its acceleration a is horizontal; hence the vertical forces balance:

$$N \cos 37^\circ = mg \;\Rightarrow\; N = \frac{2 \times 10}{0.8} = 25\ \text{N}.$$

The horizontal component $N \sin 37^\circ = 15$ N gives $a = 7.5\ \text{m s}^{-2} = g \tan 37^\circ$.

Answer: (B)

Common mistake: Writing N = mg cos θ (16 N), which holds only when the incline is not accelerating.

Q.3

Fluids – Torricelli efflux and range • Foundation • 90 s **Answer: C**

A large open tank stands on level ground and its water surface is 5.0 m above the ground. Water flows out horizontally through a small hole in the side wall at a height of 1.8 m above the ground. Treat water as ideal, assume the water level stays constant and take $g = 10\ \text{m s}^{-2}$. The jet strikes the ground at a horizontal distance from the wall of

(A) 3.6 m (B) 6.0 m (C) 4.8 m (D) 2.4 m

Solution

Depth of the hole below the free surface: $5.0 - 1.8 = 3.2$ m. By Torricelli,

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 3.2} = 8\ \text{m s}^{-1}.$$

Time to fall 1.8 m: $t = \sqrt{2 \times 1.8/10} = 0.6$ s. Range

$$x = vt = 8 \times 0.6 = 4.8\ \text{m}.$$

Answer: (C)

Common mistake: Using the height of the hole above the ground (1.8 m) or the full 5 m in Torricelli's formula instead of the depth below the free surface.

Q.4 Electrostatics – Gauss's law for a non-uniform charge distribution • Challenging • 150 s **Answer: A**

A solid non-conducting sphere of radius R carries a volume charge density $\rho(r) = \rho_0\left(1 - \frac{r}{R}\right)$, where r is the distance from its centre and $\rho_0 > 0$. Consider the magnitude of the electric field at all points inside and outside the sphere. The greatest value of the field, and the distance from the centre at which it occurs, are

(A) $\frac{\rho_0 R}{9\varepsilon_0}$ at $r = \frac{2R}{3}$

(B) $\frac{\rho_0 R}{12\varepsilon_0}$ at $r = R$

(C) $\frac{5\rho_0 R}{48\varepsilon_0}$ at $r = \frac{R}{2}$

(D) $\frac{7\rho_0 R}{64\varepsilon_0}$ at $r = \frac{3R}{4}$

Solution

Charge enclosed within radius $r \le R$:

$$q(r) = \int_0^r \rho_0\left(1 - \frac{r'}{R}\right) 4\pi r'^2\, dr' = 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^4}{4R}\right).$$

By Gauss's law, $E \cdot 4\pi r^2 = q(r)/\varepsilon_0$, so inside

$$E(r) = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right).$$

Setting $\frac{dE}{dr} = \frac{\rho_0}{\varepsilon_0}\left(\frac{1}{3} - \frac{r}{2R}\right) = 0$ gives $r = \frac{2R}{3}$ (a maximum, since $E'' < 0$), where

$$E_{\max} = \frac{\rho_0}{\varepsilon_0}\left(\frac{2R}{9} - \frac{R}{9}\right) = \frac{\rho_0 R}{9\varepsilon_0}.$$

At the surface $E(R) = \frac{\rho_0 R}{12\varepsilon_0}$, which is smaller, and outside the field falls off as $1/r^2$. So the overall maximum is $\frac{\rho_0 R}{9\varepsilon_0}$ at $r = \frac{2R}{3}$.

Answer: (A)

Common mistake: Assuming, as for a uniformly charged sphere, that the field is greatest at the surface (gives $\rho_0 R/12\varepsilon_0$ at $r = R$).

Q.5 Current electricity – cells in series with reversed cells • Foundation • 75 s **Answer: B**

A battery is made by joining 10 identical cells in series, each of emf 1.5 V and internal resistance 0.5 Ω. By mistake, two of the cells are connected with reversed polarity. The battery is connected across an external resistance of 7 Ω. The current in the circuit is

(A) 1.00 A (B) 0.75 A (C) 1.25 A (D) 0.82 A

Solution

Each reversed cell cancels one correctly connected cell, so the net emf is that of $10 - 2 \times 2 = 6$ cells:

$$E = 6 \times 1.5 = 9\ \text{V}.$$

All ten internal resistances are still in series: $r = 10 \times 0.5 = 5\ \Omega$.

$$I = \frac{E}{R + r} = \frac{9}{7 + 5} = 0.75\ \text{A}.$$

Answer: (B)

Common mistake: Subtracting only 2 cells (net 8 cells, 12 V) instead of 4, or dropping the internal resistance of the reversed cells.

Q.6 Semiconductors – Zener diode as voltage regulator • Foundation • 75 s **Answer: D**

A Zener diode of breakdown voltage 6.0 V is connected in reverse bias to a 10 V dc supply through a series resistance of 200 Ω. A load resistance of 1.0 kΩ is connected in parallel with the Zener diode. Assuming the Zener voltage stays 6.0 V in breakdown, the current through the Zener diode is

(A) 20 mA (B) 26 mA (C) 6 mA (D) 14 mA

Solution

Voltage across the series resistor: $10 - 6 = 4$ V, so

$$I_s = \frac{4}{200} = 20\ \text{mA}.$$

Load current: $I_L = \dfrac{6}{1000} = 6$ mA. Hence

$$I_Z = I_s - I_L = 14 \text{ mA}.$$

(Without the Zener the load would get $10 \times 1000/1200 = 8.3$ V > 6 V, so the Zener is indeed in breakdown.)

Answer: (D)

Common mistake: Taking the full series current (20 mA) as the Zener current and forgetting the load branch.

Q.7 Dual nature – de Broglie wavelength of accelerated particles • Foundation • 75 s **Answer: A**

A proton is accelerated from rest through a potential difference of 800 V. Through what potential difference must an α-particle (mass $4m_p$, charge $2e$) be accelerated from rest so that it has the same de Broglie wavelength as the proton?

(A) 100 V (B) 400 V (C) 200 V (D) 6400 V

Solution

For a particle of mass m and charge q accelerated from rest through V:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}.$$

Equal wavelengths need equal mqV:

$$m_p\, e\,(800) = (4m_p)(2e)\,V' \;\Rightarrow\; V' = \frac{800}{8} = 100 \text{ V}.$$

Answer: (A)

Common mistake: Accounting for only the mass ratio (200 V) or only the charge ratio (400 V) instead of the product mq.

Q.8 Work, energy and power – elastic head-on collision • Standard • 105 s **Answer: C**

A ball moving with speed v collides head-on and perfectly elastically with another ball initially at rest. In the collision the moving ball loses $\dfrac{8}{9}$ of its kinetic energy and **rebounds**. The ratio of the mass of the ball initially at rest to that of the moving ball is

(A) $\dfrac{1}{2}$ (B) 3

(C) 2 (D) 4

Solution

Let the masses be m (moving) and $M = rm$ (at rest). For an elastic head-on collision the moving ball's final velocity is

$$v_1 = \frac{m - M}{m + M}\, v = \frac{1 - r}{1 + r}\, v.$$

It keeps $\dfrac{1}{9}$ of its kinetic energy, so $|v_1| = \dfrac{v}{3}$:

$$\left|\frac{1 - r}{1 + r}\right| = \frac{1}{3} \;\Rightarrow\; r = 2 \text{ or } r = \frac{1}{2}.$$

It rebounds, so $v_1 < 0$, which needs $M > m$. Hence $r = 2$ (then $v_1 = -v/3$).

Answer: (C)

Common mistake: Picking r = 1/2, which also leaves 1/9 of the kinetic energy but makes the ball continue forward instead of rebounding.

Q.9 Rotational motion – hinge reaction at release • Standard • 150 s **Answer: C**

A uniform rod of mass m and length L is hinged at one end about a smooth horizontal axis, and a small bead of mass m is fixed at its free end. The rod is held horizontal and released from rest. At the instant of release, the magnitude of the force exerted by the hinge on the rod is

(A) $\dfrac{5mg}{32}$ (B) $\dfrac{mg}{4}$

(C) $\dfrac{5mg}{16}$ (D) $2mg$

Solution

Moment of inertia about the hinge and torque of gravity:

$$I = \frac{mL^2}{3} + mL^2 = \frac{4mL^2}{3}, \qquad \tau = mg\frac{L}{2} + mgL = \frac{3mgL}{2}.$$

$$\alpha = \frac{\tau}{I} = \frac{9g}{8L}.$$

The centre of mass of rod + bead is at $\dfrac{m(L/2)+mL}{2m} = \dfrac{3L}{4}$ from the hinge, so at release (with $\omega = 0$) its acceleration is vertical:

$$a_{cm} = \alpha \cdot \frac{3L}{4} = \frac{27g}{32}.$$

There is no horizontal reaction because $\omega = 0$. For the whole system (mass $2m$):

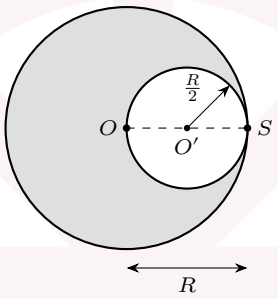
$$2mg - N = 2m \cdot \frac{27g}{32} \;\Rightarrow\; N = 2mg\left(1 - \frac{27}{32}\right) = \frac{5mg}{16}.$$

Answer: (C)

Common mistake: Quoting the bare-rod result mg/4, or using mass m instead of 2m in the centre-of-mass equation (5mg/32).

Q.10 Gravitation – field inside a spherical cavity (superposition) • Standard • 120 s **Answer: A**

A uniform solid sphere of density ρ and radius R, centre O, has a spherical cavity of radius $R/2$ whose centre O' is at distance $R/2$ from O. The cavity therefore passes through O and touches the outer surface at S, as shown. A small particle is released from rest just inside the cavity at S. Considering only the gravitational attraction of the sphere, the time the particle takes to reach O is



(A) $\sqrt{\dfrac{3}{\pi G\rho}}$ (B) $\sqrt{\dfrac{3}{2\pi G\rho}}$

(C) $\dfrac{\pi}{2}\sqrt{\dfrac{3}{4\pi G\rho}}$ (D) $\sqrt{\dfrac{6}{\pi G\rho}}$

Solution

Treat the body as a full sphere of density ρ plus a sphere of density $-\rho$ filling the cavity. Inside a uniform sphere, $\vec{g} = -\frac{4}{3}\pi G\rho\,\vec{r}$, with $\vec{r}$ measured from its centre. At a point inside the cavity:

$$\vec{g} = -\tfrac{4}{3}\pi G\rho\,\vec{r}_O + \tfrac{4}{3}\pi G\rho\,\vec{r}_{O'} = -\tfrac{4}{3}\pi G\rho\,\overrightarrow{OO'}.$$

So the field in the cavity is **uniform**, directed from O' towards O (along SO), with magnitude

$$g = \tfrac{4}{3}\pi G\rho \cdot \frac{R}{2} = \tfrac{2}{3}\pi G\rho R.$$

Released from rest at S, the particle moves in a straight line along the diameter SO (length R, entirely inside the cavity) with constant acceleration g:

$$R = \tfrac{1}{2}gt^2 \;\Rightarrow\; t = \sqrt{\frac{2R}{\frac{2}{3}\pi G\rho R}} = \sqrt{\frac{3}{\pi G\rho}}.$$

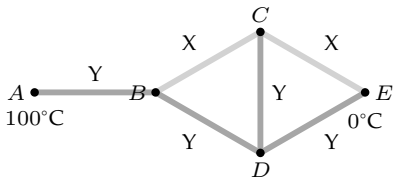
Answer: (A)

Common mistake: Treating the motion as SHM through a solid sphere (quarter period), or using the field at the surface of the full sphere (4πGρR/3), instead of the uniform cavity field (4/3)πGρ·OO′.

Q.11

Heat transfer – thermal resistance network (balanced bridge) • Standard • 120 s **Answer: B**

Six rods of identical length L and cross-sectional area A are joined as shown in the figure. Rods BC and CE are made of material X (thermal conductivity K). Rods AB, BD, DE and CD are made of material Y (thermal conductivity $3K$). End A is kept at 100°C and junction E at 0°C. The rods are insulated along their lengths and the steady state is reached. The temperature of junction B is



(A) 50°C

(B) 60°C

(C) $\frac{100}{3}^\circ$C

(D) 40°C

Solution

Let $R = \dfrac{L}{KA}$. Then each X rod has resistance R and each Y rod has $\dfrac{R}{3}$.

Bridge B–C–E / B–D–E with CD across it:

$$\frac{R_{BC}}{R_{CE}} = \frac{R}{R} = 1 = \frac{R_{BD}}{R_{DE}} = \frac{R/3}{R/3},$$

so it is balanced: C and D are at the same temperature and no heat flows in CD.

$$R_{BE} = (2R) \parallel \left(\frac{2R}{3}\right) = \frac{2R \cdot \frac{2R}{3}}{2R + \frac{2R}{3}} = \frac{R}{2}, \qquad R_{AE} = \frac{R}{3} + \frac{R}{2} = \frac{5R}{6}.$$

The heat current is the same through AB and the rest, so the temperature drop divides in proportion to resistance:

$$T_A - T_B = 100 \times \frac{R/3}{5R/6} = 40^\circ\text{C} \;\Rightarrow\; T_B = 60^\circ\text{C}.$$

Answer: (B)

Common mistake: Taking the resistance of a rod as proportional to its conductivity (R ∝ K), which gives 100/3 °C, or reporting the 40 °C drop as the temperature of B.

Q.12

Simple harmonic motion – mass added at mean position • Standard • 105 s **Answer: D**

A block of mass M attached to a horizontal spring oscillates on a smooth horizontal floor with amplitude A. As the block passes through its mean position, a lump of putty of mass $3M$ is dropped vertically onto it and sticks at once. The new amplitude of oscillation is

(A) $\frac{A}{4}$

(B) $\frac{A}{\sqrt{2}}$

(C) A

(D) $\frac{A}{2}$

Solution

At the mean position the block has speed $v = A\omega$, with $\omega = \sqrt{k/M}$. The putty brings no horizontal momentum, so horizontal momentum is conserved:

$$Mv = 4M\,v' \;\Rightarrow\; v' = \frac{v}{4}.$$

The new angular frequency is $\omega' = \sqrt{k/4M} = \omega/2$. The block is still at the mean position, so

$$A' = \frac{v'}{\omega'} = \frac{v/4}{\omega/2} = \frac{A}{2}.$$

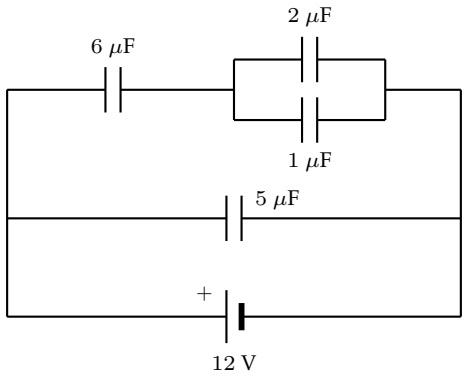
Answer: (D)

Common mistake: Dividing the new speed by the old ω (gives A/4), forgetting that adding mass also lowers the angular frequency.

Q.13

Capacitance – series-parallel combination • Standard • 90 s **Answer: B**

In the circuit shown, all capacitors are initially uncharged and the 12 V battery is ideal. In the steady state, the charge on the 1 μF capacitor is



(A) 16 μC (B) 8 μC (C) 24 μC (D) 4 μC

Solution

All three branches are connected across the same two rails, so each branch has 12 V across it; the 5 μF branch does not affect the top branch.

Top branch: 2 μF $\parallel$ 1 μF $= 3$ μF, in series with 6 μF:

$$C_{\text{top}} = \frac{6 \times 3}{6+3} = 2\ \mu\text{F}, \qquad Q = 2 \times 12 = 24\ \mu\text{C}.$$

This charge sits on the 6 μF and on the parallel pair. Voltage across the pair:

$$V = \frac{24}{3} = 8\ \text{V} \ \Rightarrow\ Q_{1\,\mu\text{F}} = 1 \times 8 = 8\ \mu\text{C}.$$

(The 2 μF holds 16 μC.)

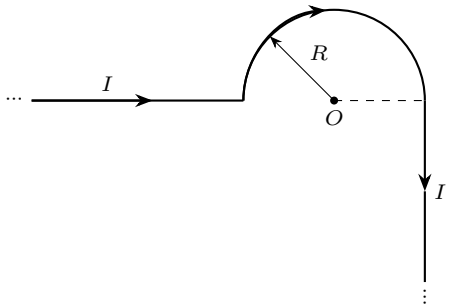
Answer: (B)

Common mistake: Giving the full series charge 24 μC to the 1 μF capacitor instead of splitting it between the parallel pair.

Q.14

Magnetic effects of current – Biot–Savart for a bent wire • Standard • 120 s **Answer: C**

A very long wire carrying current I is bent into the shape shown and lies in the plane of the page. A straight part comes in along a line passing through O and stops at distance R from O; it continues as a semicircle of radius R centred at O; it then leaves along a straight line perpendicular to the first part, tangent to the semicircle at its other end, and goes to infinity. The magnitude of the magnetic field at O is



(A) $\dfrac{\mu_0 I}{4R}$ (B) $\dfrac{\mu_0 I}{4\pi R}(\pi+2)$

(C) $\dfrac{\mu_0 I}{4\pi R}(\pi+1)$ (D) $\dfrac{\mu_0 I}{4\pi R}(\pi-1)$

Solution

Take the three parts one at a time.

- Incoming straight part: its line passes through O, so $d\vec{l} \parallel \vec{r}$ and its field at O is zero.

- Semicircle: $B_1 = \dfrac{\mu_0 I}{4R}$; the current runs clockwise in the figure, so $\vec{B}_1$ points into the page.
- Outgoing straight part: it is semi-infinite, and O lies on the perpendicular through its end at distance R:

$$B_2 = \frac{\mu_0 I}{4\pi R}.$$

The current flows downward on the right of O, so $\vec{B}_2$ also points into the page.

$$B = \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R} = \frac{\mu_0 I}{4\pi R}(\pi + 1).$$

Answer: (C)

Common mistake: Counting the radial straight part as a semi-infinite wire (giving $\pi + 2$), or subtracting the two contributions without checking their directions.

Q.15

Alternating current – choke coil in series with a bulb • Standard • 105 s **Answer: A**

A bulb rated 100 V, 50 W has to be operated at its rated values on a 200 V (rms), 50 Hz ac supply. A pure inductor is connected in series with it for this purpose. Treating the bulb as a pure resistor, the inductance needed is

(A) $\dfrac{2\sqrt{3}}{\pi}$ H (B) $\dfrac{4}{\pi}$ H

(C) $\dfrac{\sqrt{3}}{\pi}$ H (D) $\dfrac{2}{\pi}$ H

Solution

Bulb: $R = \dfrac{V^2}{P} = \dfrac{100^2}{50} = 200\ \Omega$, rated current $I = \dfrac{50}{100} = 0.5$ A.

Required impedance: $Z = \dfrac{200}{0.5} = 400\ \Omega$. Then

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{400^2 - 200^2} = 200\sqrt{3}\ \Omega,$$

$$L = \frac{X_L}{2\pi f} = \frac{200\sqrt{3}}{100\pi} = \frac{2\sqrt{3}}{\pi}\ \text{H}\ (\approx 1.1\ \text{H}).$$

Answer: (A)

Common mistake: Adding voltages arithmetically (V_L = 100 V, so X_L = 200 Ω and L = 2/π H) instead of in quadrature.

Q.16

Experimental skills – error in focal length (u–v method) • Standard • 120 s **Answer: D**

In the u–v method for a convex lens, a student measures the object distance as (30.0 ± 0.1) cm and the image distance as (60.0 ± 0.2) cm (both magnitudes; the image is real). The focal length is found from $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ (magnitudes). The maximum possible absolute error in f, found by differentiating this relation, is closest to

(A) 0.30 cm (B) 0.10 cm (C) 0.13 cm (D) 0.067 cm

Solution

$f = \dfrac{uv}{u+v} = \dfrac{30 \times 60}{90} = 20$ cm.

Differentiate $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ and add the magnitudes for the maximum error:

$$\frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} = \frac{0.1}{900} + \frac{0.2}{3600} = \frac{1}{6000}.$$

$$\Delta f = \frac{400}{6000} = \frac{1}{15}\ \text{cm} \approx 0.067\ \text{cm}.$$

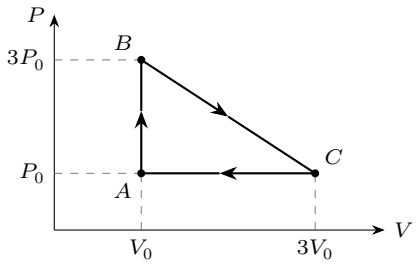
Answer: (D)

Common mistake: Adding the absolute errors Δu + Δv directly (0.30 cm), or using Δf/f = Δu/u + Δv/v (0.13 cm), instead of differentiating 1/f = 1/u + 1/v, which gives $\Delta f/f^2 = \Delta u/u^2 + \Delta v/v^2$.

Q.17

Thermodynamics – efficiency of a cycle with a straight-line process • Challenging • 210 s **Answer: C**

One mole of a monatomic ideal gas is taken through the cycle $A \to B \to C \to A$ shown on the P–V diagram: $A(V_0, P_0)$, $B(V_0, 3P_0)$ and $C(3V_0, P_0)$, where BC is a straight line. The efficiency of the cycle is



(A) $\frac{2}{7}$ (B) $\frac{2}{3}$

(C) $\frac{4}{15}$ (D) $\frac{1}{5}$

Solution

Work done per cycle = area of the triangle: $W = \frac{1}{2}(2V_0)(2P_0) = 2P_0V_0$.

$A \to B$ (isochoric): $Q_{AB} = \frac{3}{2}\,\Delta(PV) = \frac{3}{2}(3-1)P_0V_0 = 3P_0V_0$ (absorbed).

$C \to A$ (isobaric compression): heat is rejected.

$B \to C$: in units of P_0, V_0, the line is $P = 4 - V$. Along it,

$$dQ = \tfrac{3}{2}\,d(PV) + P\,dV = \left[\tfrac{3}{2}(4-2V) + (4-V)\right]dV = (10-4V)\,dV.$$

So heat is absorbed only up to $V = 2.5V_0$ and rejected after that:

$$Q_{B\to 2.5} = \int_1^{2.5} (10-4V)\,dV = 15 - 10.5 = 4.5\,P_0V_0, \qquad Q_{2.5\to C} = -0.5\,P_0V_0.$$

Total heat absorbed $= 3 + 4.5 = 7.5\,P_0V_0$:

$$\eta = \frac{W}{Q_{\text{in}}} = \frac{2}{7.5} = \frac{4}{15}.$$

(Check: $Q_{CA} = \frac{5}{2}(1-3)P_0V_0 = -5P_0V_0$; the net is $3 + 4 - 5 = 2 = W$.)

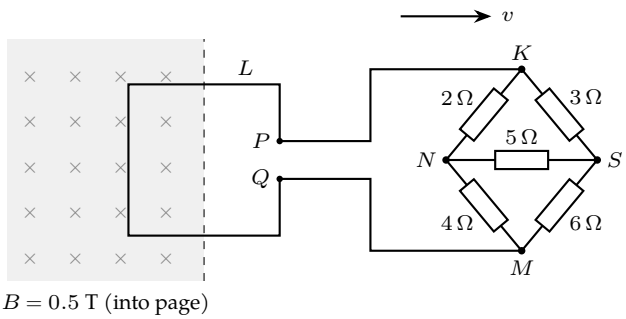
Answer: (C)

Common mistake: Treating the whole of BC as heat-absorbing ($Q_{BC} = +4P_0V_0$, giving 2/7); the sign of dQ changes in the middle of BC.

Q.18

Electromagnetic induction – motional emf driving a bridge network • Challenging • 180 s **Answer: D**

A square conducting loop L of side 10 cm has a small gap PQ in its right side; its wire has a total resistance of $1.4\ \Omega$. Terminals P and Q are joined by resistanceless wires to nodes K and M of the resistor network shown. The whole arrangement moves to the right with a constant speed $v = 5$ cm s^{-1}. At the instant shown, the left side of L is inside a uniform magnetic field of 0.5 T directed into the page, while the right side of L, the connecting wires and the network are outside the field. Neglect self-inductance. The current in the loop at this instant is



(A) 250 μA (B) 694 μA (C) 1786 μA (D) 500 μA

Solution

Only the left side of L (length 10 cm) moves through the field across its length, so

$$\varepsilon = B\ell v = 0.5 \times 0.10 \times 0.05 = 2.5 \times 10^{-3}\ \text{V}.$$

The top and bottom sides move along their own length and have no emf.

Network between K and M:

$$\frac{R_{KN}}{R_{NM}} = \frac{2}{4} = \frac{1}{2} = \frac{3}{6} = \frac{R_{KS}}{R_{SM}},$$

so the bridge is balanced and the $5\ \Omega$ carries no current:

$$R_{KM} = (2+4) \parallel (3+6) = \frac{6 \times 9}{15} = 3.6\ \Omega.$$

The loop wire is in series with the network:

$$I = \frac{2.5 \times 10^{-3}}{1.4 + 3.6} = 5 \times 10^{-4}\ \text{A} = 500\ \mu\text{A}.$$

Answer: (D)

Common mistake: Leaving out the loop's own resistance (gives 694 μA) or failing to spot the balanced bridge.

Q.19

Current electricity – potentiometer: emf and internal resistance • Challenging • 180 s **Answer: A**

A potentiometer wire AB is 400 cm long and has a resistance of $8\ \Omega$. It is connected in series with a $4\ \Omega$ resistor and a driver cell of emf 2.4 V (negligible internal resistance). A cell of emf E and internal resistance r is balanced at 250 cm from A when it is on open circuit. When a $6\ \Omega$ resistor is connected across this cell, the balance point shifts to 200 cm. Then (E, r) is

(A) $(1.0\ \text{V},\ 1.5\ \Omega)$ (B) $(1.0\ \text{V},\ 1.2\ \Omega)$ (C) $(1.5\ \text{V},\ 1.5\ \Omega)$ (D) $(1.5\ \text{V},\ 1.2\ \Omega)$

Solution

Current in the potentiometer wire: $I = \dfrac{2.4}{8+4} = 0.2$ A. Potential gradient:

$$k = \frac{0.2 \times 8}{400} = 0.004\ \text{V cm}^{-1}.$$

Open-circuit balance gives the emf: $E = 250 \times 0.004 = 1.0$ V.

With the shunt $S = 6\ \Omega$, the balance gives the terminal voltage $V = 200k$, and $E/V = (S+r)/S$:

$$r = S\left(\frac{l_1}{l_2} - 1\right) = 6\left(\frac{250}{200} - 1\right) = 1.5\ \Omega.$$

Answer: (A)

Common mistake: Ignoring the 4 Ω series resistor (takes the gradient as 2.4/400, giving E = 1.5 V), or using $r = S(l_1 - l_2)/l_1$ (gives 1.2 Ω).

Q.20

Wave optics – YDSE with a thin sheet, apparatus immersed in water • Challenging • 180 s **Answer: B**

In a Young's double-slit experiment in air, a thin glass sheet of refractive index 1.5 placed over one slit shifts the central bright fringe to the position previously occupied by the 5^{th} bright fringe. The whole apparatus, with the sheet still in place, is now immersed in water (refractive index 4/3). In water, the central bright fringe is displaced from the centre of the screen by how many fringe widths? (Fringe width measured in water.)

(A) 5 (B) $\dfrac{5}{3}$

(C) $\dfrac{5}{4}$ (D) $\dfrac{20}{3}$

Solution

In air the path difference introduced by the sheet is $(\mu_g - 1)t = 5\lambda$, so $t = 10\lambda$ (λ = wavelength in air).

In water the central fringe lies where the optical path difference is zero. The sheet adds an extra optical path $(\mu_g - \mu_w)t$. Fringe widths in water correspond to steps of $\lambda_w = \lambda/\mu_w$ in geometric path, that is, λ in optical path. So the number of fringe widths is

$$n = \frac{(\mu_g - \mu_w)\,t}{\lambda} = \left(1.5 - \frac{4}{3}\right) \times 10 = \frac{10}{6} = \frac{5}{3}.$$

(Equivalently: shift $= \left(\dfrac{\mu_g}{\mu_w} - 1\right)\dfrac{tD}{d} = \dfrac{5}{4}\beta_{\text{air}}$ and $\beta_w = \dfrac{3}{4}\beta_{\text{air}}$, giving $\dfrac{5}{3}$.)

Answer: (B)

Common mistake: Using the relative index μ_g/μ_w but forgetting that the fringe width also shrinks in water (gives 5/4).

Q.21 Thermodynamics – heat capacities of a gas mixture (isobaric then isochoric) • Standard • 150 s **Answer: 50**

A cylinder with a frictionless, movable piston contains a mixture of 4 g of helium and 28 g of nitrogen. Treat both as ideal gases, with nitrogen rigid diatomic (no vibration). First, 1000 J of heat is supplied slowly with the piston free, so the pressure stays constant. The piston is then clamped and a further 1000 J of heat is supplied. There is no heat loss. Take $R = \frac{25}{3}$ J mol^{-1} K^{-1}. Find the total rise in temperature of the mixture, in kelvin.

Solution

Moles: $n_{\mathrm{He}} = 4/4 = 1$, $n_{\mathrm{N_2}} = 28/28 = 1$, so $n = 2$.

Mixture:

$$C_V = \frac{1 \cdot \frac{3}{2}R + 1 \cdot \frac{5}{2}R}{2} = 2R, \qquad C_P = C_V + R = 3R.$$

Stage 1 (isobaric):

$$\Delta T_1 = \frac{Q}{nC_P} = \frac{1000}{2 \times 3 \times \frac{25}{3}} = 20 \text{ K}.$$

Stage 2 (isochoric, piston clamped):

$$\Delta T_2 = \frac{Q}{nC_V} = \frac{1000}{2 \times 2 \times \frac{25}{3}} = 30 \text{ K}.$$

$$\Delta T = 20 + 30 = 50 \text{ K}.$$

Answer: 50

Common mistake: Using C_V for both stages (gives 60 K), or averaging the heat capacities by mass instead of by moles.

Q.22 Rotational motion – impulse giving immediate pure rolling • Standard • 120 s **Answer: 14**

A uniform solid sphere of radius 10 cm rests on a rough horizontal floor. It is given a sharp horizontal impulse whose line of action lies in the vertical plane through the centre, at height h above the floor. Find h (in cm) for which the sphere starts rolling without slipping immediately, so that friction plays no role.

Solution

With no friction impulse, $J = mv$ and the angular impulse about the centre is $J(h - R) = I\omega = \frac{2}{5}mR^2\omega$.

For pure rolling at once, $v = \omega R$:

$$mv(h - R) = \frac{2}{5}mR^2 \cdot \frac{v}{R} \;\Rightarrow\; h - R = \frac{2R}{5} \;\Rightarrow\; h = \frac{7R}{5} = 14 \text{ cm}.$$

Answer: 14

Common mistake: Measuring h from the centre instead of from the floor (answer 4), or using the ring/disc moment of inertia.

Q.23 Ray optics – lens–convex mirror combination (image on object) • Standard • 120 s **Answer: 10**

A point object is placed on the principal axis of a thin convex lens of focal length 12 cm, at a distance of 18 cm from it. A convex mirror is placed coaxially on the other side of the lens, 16 cm from the lens. The final image formed by the system coincides with the object itself. Find the magnitude of the focal length of the convex mirror, in cm.

Solution

Lens alone ($u = -18$, $f = +12$):

$$\frac{1}{v} = \frac{1}{12} - \frac{1}{18} = \frac{1}{36} \;\Rightarrow\; v = 36 \text{ cm}.$$

The converging rays head for a point $36 - 16 = 20$ cm behind the mirror. The image coincides with the object only if the rays retrace their path, that is, if they strike the mirror normally. So they must be directed toward the centre of curvature:

$$R = 20 \text{ cm}, \qquad |f| = \frac{R}{2} = 10 \text{ cm}.$$

Answer: 10

Common mistake: Taking the 20 cm as the focal length instead of the radius of curvature.

Q.24

Nuclei – activity of two radioactive species • Standard • 120 s **Answer: 60**

A sample initially contains equal numbers of nuclei of two radioactive species A (half-life 10 min) and B (half-life 20 min); neither decays into the other. After how many minutes will the activity of A become one-fourth of the activity of B?

Solution

Activity $= \lambda N$ with $\lambda = \ln 2/T_{1/2}$, so $\lambda_A = 2\lambda_B$. With $N_A(0) = N_B(0) = N_0$:

$$\frac{R_A}{R_B} = \frac{\lambda_A N_0 2^{-t/10}}{\lambda_B N_0 2^{-t/20}} = 2 \cdot 2^{-t/20}.$$

Setting this equal to $\frac{1}{4}$: $2^{-t/20} = \frac{1}{8} \Rightarrow t/20 = 3 \Rightarrow t = 60$ min.

Answer: 60

Common mistake: Comparing numbers of nuclei instead of activities (gives 40 min), forgetting that A starts with twice B's activity.

Q.25

Waves – Doppler effect with reflection and beats • Challenging • 180 s **Answer: 112**

A car moves at 30 m s^{-1} straight towards a large vertical wall, sounding a horn of frequency 600 Hz. A cyclist behind the car, on the same straight line, rides towards the wall at 6 m s^{-1} (the car is between the cyclist and the wall). The cyclist hears the sound coming directly from the car and the sound reflected from the wall. Speed of sound in still air $= 330$ m s^{-1}; there is no wind. Find the beat frequency (in Hz) heard by the cyclist.

Solution

Direct sound. The source moves away from the cyclist and the cyclist moves towards the source:

$$f_1 = 600 \cdot \frac{330 + 6}{330 + 30} = 600 \cdot \frac{336}{360} = 560 \text{ Hz}.$$

Reflected sound. The wall (stationary) receives from the approaching car

$$f_w = 600 \cdot \frac{330}{330 - 30} = 660 \text{ Hz},$$

and re-emits it. The cyclist approaches this stationary source:

$$f_2 = 660 \cdot \frac{330 + 6}{330} = 672 \text{ Hz}.$$

$$f_{\text{beat}} = f_2 - f_1 = 672 - 560 = 112 \text{ Hz}.$$

Answer: 112

Common mistake: Ignoring the cyclist's motion for the reflected sound (gives 100 Hz), or treating the car as approaching the cyclist for the direct sound.

PART B : CHEMISTRY — Detailed solutions

Q.26

Isomerism – geometrical isomerism in alkenes • Foundation • 45 s **Answer: C**

Which one of the following alkenes does **not** show geometrical (cis–trans) isomerism?

(A) BrCH=CHCl
(B) $CH_3CH{=}C(CH_3)COOH$
(C) $CH_3CH{=}C(CH_2CH_3)_2$
(D) $CH_3CH{=}C(CH_3)CH_2CH_3$

Solution

Geometrical isomerism needs **each** doubly bonded carbon to carry two different groups.

- (A) BrCH=CHCl: C1 has Br, H; C2 has Cl, H. Both carbons are unsymmetrical, so cis and trans forms exist.
- (B) $CH_3CH{=}C(CH_3)COOH$: C2 has CH_3, H; C3 has CH_3, COOH. Isomers exist (tiglic and angelic acids).
- (C) $CH_3CH{=}C(CH_2CH_3)_2$ (3-ethylpent-2-ene): C2 has CH_3, H, but C3 carries **two ethyl groups**. Swapping them gives the same molecule, so there is no cis–trans isomerism.
- (D) $CH_3CH{=}C(CH_3)CH_2CH_3$ (3-methylpent-2-ene): C3 has CH_3 and C_2H_5, which are different, so E and Z forms exist.

Answer: (C)

Common mistake: Looking only at the longest chain and missing that C3 in 3-ethylpent-2-ene carries two identical ethyl groups.

Q.27

Periodicity – trends in ionisation enthalpy, radius and electron gain enthalpy (with exceptions) • Standard • 90 s **Answer: A**

In which option are **all three** orders correct? (Size means atomic radius for atoms and ionic radius for ions.)

(A) First ionisation enthalpy: B < C < O < N; size: Ga < Al < In; magnitude of electron gain enthalpy: O < S < Cl
(B) First ionisation enthalpy: Na < Al < Mg; size: Ga < Al < In; magnitude of electron gain enthalpy: Br < Cl < F
(C) First ionisation enthalpy: Be < B < C; size: Ga < Al < In; magnitude of electron gain enthalpy: O < S < Cl
(D) First ionisation enthalpy: B < C < O < N; size: $Na^+ < Mg^{2+} < Al^{3+}$; magnitude of electron gain enthalpy: O < S < Cl

Solution

Check each order against the NCERT data and the known exceptions.

- **Ionisation enthalpy, B < C < O < N:** correct (801, 1086, 1314 and 1402 kJ mol^{-1}). Two exceptions apply. B (whose outer electron is $2p^1$) lies below Be, whose $2s$ electron penetrates better. O lies below N because N has a stable half-filled $2p^3$ subshell, while O loses a paired $2p$ electron.
- **Ionisation enthalpy, Na < Al < Mg:** correct (496, 577 and 738). Al ($3p^1$) lies below Mg ($3s^2$).
- **Ionisation enthalpy, Be < B < C: wrong**, because Be (899) > B (801).
- **Atomic radius, Ga < Al < In:** correct (135, 143 and 167 pm). Ga is smaller than Al because its ten poorly shielding $3d$ electrons raise the effective nuclear charge.
- **Ionic radius, $Na^+ < Mg^{2+} < Al^{3+}$: wrong**. These ions are isoelectronic, and the radius falls as the nuclear charge rises: $Na^+ > Mg^{2+} > Al^{3+}$.
- **Electron gain enthalpy, O < S < Cl:** correct (141, 200 and 349 in magnitude). O lies below S because its small $2p$ shell causes strong electron–electron repulsion.
- **Electron gain enthalpy, Br < Cl < F: wrong**. F (328) lies below Cl (349) for the same small-size reason.

Only option (A) has all three orders correct. Options (B), (C) and (D) each contain exactly one wrong order.

Answer: (A)

Common mistake: Applying the smooth period and group trends without the exceptions, e.g. taking Ga larger than Al or F above Cl in electron gain enthalpy.

Q.28

Chemical bonding – VSEPR and bond angles of hydrides • Foundation • 50 s **Answer: D**

The correct decreasing order of the H–X–H bond angle in H_2O, NH_3, PH_3 and H_2S is

(A) $NH_3 > H_2O > H_2S > PH_3$
(B) $H_2O > NH_3 > H_2S > PH_3$
(C) $NH_3 > PH_3 > H_2O > H_2S$
(D) $NH_3 > H_2O > PH_3 > H_2S$

Solution

- **NH_3 vs H_2O:** both central atoms are roughly sp^3. NH_3 has one lone pair and H_2O has two; more lone-pair repulsion squeezes the angle more. So NH_3 (107.8°) > H_2O (104.5°).

- **Third-period hydrides:** P and S are larger and less electronegative, and their bonds use almost pure p orbitals, so the angles fall towards 90°: PH_3 (93.6°) > H_2S (92.1°). Again the hydride with fewer lone pairs (PH_3) has the larger angle.

$$NH_3 > H_2O > PH_3 > H_2S$$

Answer: (D)

Common mistake: Placing PH_3 above H_2O because both are group-15/16 'pyramidal vs bent', ignoring the drop to near-90° angles in the third period.

Q.29 Hydrocarbons – Friedel–Crafts alkylation and rearrangement • Foundation • 45 s **Answer: B**

Benzene is treated with 1-chloropropane, $CH_3CH_2CH_2Cl$, in the presence of anhydrous $AlCl_3$. The major product is

(A) n-Propylbenzene, $C_6H_5CH_2CH_2CH_3$
(B) Isopropylbenzene (cumene), $C_6H_5CH(CH_3)_2$
(C) Ethylbenzene, $C_6H_5CH_2CH_3$
(D) tert-Butylbenzene, $C_6H_5C(CH_3)_3$

Solution

$AlCl_3$ generates an electrophile from the alkyl halide. The primary n-propyl cation (or the polarised $CH_3CH_2CH_2^{\delta+}\cdots Cl-AlCl_3$ complex) undergoes a **1,2-hydride shift** to the more stable secondary isopropyl cation:

$$CH_3CH_2CH_2^{+} \longrightarrow CH_3-\overset{+}{C}H-CH_3$$

The isopropyl cation then attacks benzene, so the major product is isopropylbenzene (cumene). n-Propylbenzene is only a minor product; options (C) and (D) have the wrong number of carbon atoms.

Answer: (B)

Common mistake: Writing n-propylbenzene by simply joining the propyl chain to the ring, forgetting the carbocation rearrangement.

Q.30 d-Block – oxidising action of KMnO4 and K2Cr2O7 • Foundation • 50 s **Answer: A**

Which of the following statements is **incorrect**?

(A) In neutral or faintly alkaline solution, $KMnO_4$ oxidises iodide ions to iodine.
(B) Acidified $K_2Cr_2O_7$ oxidises iodide ions to iodine.
(C) Acidified $KMnO_4$ oxidises oxalate ions to CO_2 at about 333 K.
(D) On acidification, an aqueous chromate solution changes to dichromate: $2\,CrO_4^{2-} + 2\,H^+ \longrightarrow Cr_2O_7^{2-} + H_2O$.

Solution

- (A) **False.** In neutral or faintly alkaline medium, MnO_4^- is reduced to MnO_2 and iodide is oxidised further, to **iodate**: $2\,MnO_4^- + H_2O + I^- \longrightarrow 2\,MnO_2 + 2\,OH^- + IO_3^-$. Iodide gives I_2 only in acidic medium.
- (B) True. $Cr_2O_7^{2-} + 14\,H^+ + 6\,I^- \longrightarrow 2\,Cr^{3+} + 3\,I_2 + 7\,H_2O$.
- (C) True. NCERT: in acid, MnO_4^- oxidises oxalate (or oxalic acid) to CO_2 at 333 K.
- (D) True. The chromate–dichromate interconversion depends on pH; acid favours dichromate.

Answer: (A)

Common mistake: Assuming permanganate always oxidises I^- to I_2, whatever the medium.

Q.31 Biomolecules – protein denaturation and starch • Foundation • 45 s **Answer: C**

Given below are two statements.

Statement I: When a globular protein is denatured, for example by heating, its primary structure is destroyed.

Statement II: Amylose, which makes up about 15–20% of starch, is the water-soluble component of starch.

Choose the correct option.

(A) Both Statement I and Statement II are false
(B) Both Statement I and Statement II are true
(C) Statement I is false but Statement II is true
(D) Statement I is true but Statement II is false

Solution

Statement I: False. During denaturation, the hydrogen bonds and other forces that hold the secondary and tertiary structures are disturbed: the globules unfold, helices uncoil and biological activity is lost. The peptide bonds are not broken, so the **primary structure remains intact** (NCERT).

Statement II: True. Starch consists of amylose (water-soluble, about 15–20%) and amylopectin (water-insoluble, about 80–85%).

So Statement I is false and Statement II is true.

Answer: (C)

Common mistake: Thinking denaturation breaks peptide bonds; it destroys only the 2° and 3° structures.

Q.32 Alcohols – carbocation rearrangement in substitution and dehydration • Challenging • 120 s **Answer: B**

2,2-Dimethylpentan-3-ol, $(CH_3)_3C-CH(OH)-CH_2CH_3$, is subjected to two separate reactions.

P: heating with concentrated HBr.

Q: heating with concentrated H_2SO_4.

The major organic products of P and Q, respectively, are

(A) 3-Bromo-2,2-dimethylpentane and 4,4-dimethylpent-2-ene

(B) 2-Bromo-2,3-dimethylpentane and 2,3-dimethylpent-2-ene

(C) 2-Bromo-2,3-dimethylpentane and 2,3-dimethylpent-1-ene

(D) 3-Bromo-2,2-dimethylpentane and 2,3-dimethylpent-2-ene

Solution

Both reactions start in the same way: the –OH is protonated and leaves as water. The C3 carbocation formed is secondary and next to a quaternary carbon, so both reactions go through carbocations (S_N1 and E1). A direct S_N2 attack at C3 is strongly hindered by the neighbouring tert-butyl group.

Rearrangement: a **1,2-methyl shift** from C2 to C3 turns the secondary cation into a more stable tertiary cation:

$$(CH_3)_3C-\overset{+}{C}H-CH_2CH_3 \longrightarrow (CH_3)_2\overset{+}{C}-CH(CH_3)-CH_2CH_3$$

- **P:** Br^- captures the tertiary cation, giving $(CH_3)_2CBr-CH(CH_3)CH_2CH_3$, which is **2-bromo-2,3-dimethylpentane**.
- **Q:** the cation loses a β-hydrogen. Losing it from C3 gives the tetrasubstituted alkene $(CH_3)_2C=C(CH_3)CH_2CH_3$, **2,3-dimethylpent-2-ene**. By Saytzeff's rule this is the major product. Losing it from a methyl group gives the less substituted 2,3-dimethylpent-1-ene, which is only a minor product.

The unrearranged secondary cation would give 3-bromo-2,2-dimethylpentane and 4,4-dimethylpent-2-ene; these appear in options (A) and (D).

Answer: (B)

Common mistake: Writing the product without the 1,2-methyl shift (3-bromo-2,2-dimethylpentane), or choosing the less substituted terminal alkene in the dehydration.

Q.33 Amines – Hinsberg test • Foundation • 60 s **Answer: D**

An amine X, C_3H_9N, reacts with benzenesulphonyl chloride to give a product that is **insoluble** in aqueous KOH. X is

(A) Propan-1-amine

(B) N,N-Dimethylmethanamine

(C) Propan-2-amine

(D) N-Methylethanamine

Solution

Hinsberg's reagent ($C_6H_5SO_2Cl$):

- A **primary** amine gives $C_6H_5SO_2NHR$. The H on N is acidic, so the sulphonamide dissolves in KOH.
- A **secondary** amine gives $C_6H_5SO_2NR_2$, which has no N–H and is insoluble in KOH.
- A **tertiary** amine does not react.

The isomers of C_3H_9N are propan-1-amine and propan-2-amine (1°), N-methylethanamine $CH_3NHC_2H_5$ (2°), and trimethylamine (3°). Only the secondary amine fits.

Answer: (D)

Common mistake: Choosing trimethylamine: a tertiary amine gives no sulphonamide at all, so there is no 'insoluble product'.

Q.34 Atomic structure – Bohr model for hydrogen-like ions • Standard • 100 s **Answer: D**

In a hydrogen-like ion, the photon emitted in the transition $n = 3 \to n = 2$ has energy 68.0 eV. The minimum energy needed to remove the electron from the $n = 2$ level of this ion is (take the ground-state energy of H as -13.6 eV)

(A) 30.6 eV (B) 489.6 eV (C) 54.4 eV (D) 122.4 eV

Solution

For a hydrogen-like ion, $E_n = -\dfrac{13.6\,Z^2}{n^2}$ eV.

Step 1 (find Z):

$$\Delta E_{3\to2} = 13.6\,Z^2\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{13.6\times5}{36}Z^2 = 68.0 \;\Rightarrow\; Z^2 = 36,\; Z = 6\;(\mathrm{C}^{5+})$$

Step 2 (ionise from n = 2):

$$E = \frac{13.6\times36}{4} = 122.4\ \text{eV}$$

Option (B) is the ionisation energy from $n = 1$, and (C) uses $Z = 2$.

Answer: (D)

Common mistake: Answering with the ground-state ionisation energy $13.6Z^2 = 489.6$ eV instead of removing the electron from $n = 2$.

Q.35 Redox – n-factor and equivalents (dichromate titration) • Standard • 100 s **Answer: A**

What mass of iron(II) oxalate, FeC_2O_4, is completely oxidised by 50.0 mL of 0.100 M $K_2Cr_2O_7$ in acidic medium? (Fe = 56, C = 12, O = 16; the products are Fe^{3+}, CO_2 and Cr^{3+}.)

(A) 1.44 g (B) 2.16 g (C) 0.72 g (D) 4.32 g

Solution

Electrons accepted by dichromate: $Cr_2O_7^{2-} \longrightarrow 2\,Cr^{3+}$ takes 6 electrons per ion.

$$n(e^-) = 0.0500\times0.100\times6 = 0.0300\ \text{mol}$$

n-factor of FeC_2O_4: $Fe^{2+} \longrightarrow Fe^{3+}$ loses 1 electron, and $C_2O_4^{2-} \longrightarrow 2\,CO_2$ loses 2 electrons. The total is 3.

$$n(FeC_2O_4) = \frac{0.0300}{3} = 0.0100\ \text{mol}$$

Molar mass $= 56 + 24 + 64 = 144\ \text{g mol}^{-1}$, so the mass $= 0.0100\times144 = 1.44$ g.

Option (D) counts only the Fe^{2+} oxidation (n-factor 1), and (B) counts only the oxalate (n-factor 2).

Answer: (A)

Common mistake: Taking the n-factor of FeC_2O_4 as 1 or 2: both the iron and the oxalate are oxidised, so it is 3.

Q.36 Chemical equilibrium – degree of dissociation from vapour density • Standard • 110 s **Answer: C**

For $N_2O_4(g) \rightleftharpoons 2\,NO_2(g)$ at a certain temperature, the equilibrium mixture at a total pressure of 2.0 atm has a vapour density of 34.5 (relative to H_2). The value of K_p at this temperature is (N = 14, O = 16)

(A) 0.50 atm (B) 2.0 atm (C) 1.0 atm (D) 0.25 atm

Solution

Mean molar mass of the mixture $= 2\times34.5 = 69\ \text{g mol}^{-1}$; $M(N_2O_4) = 92$.

Starting from 1 mol N_2O_4 with degree of dissociation α, the total amount is $1+\alpha$:

$$\frac{92}{1+\alpha} = 69 \;\Rightarrow\; \alpha = \frac{1}{3}$$

Mole fractions: $x_{N_2O_4} = \dfrac{1-\alpha}{1+\alpha} = \dfrac{1}{2}$ and $x_{NO_2} = \dfrac{2\alpha}{1+\alpha} = \dfrac{1}{2}$. So $p_{N_2O_4} = p_{NO_2} = 1.0$ atm.

$$K_p = \frac{(1.0)^2}{1.0} = 1.0\ \text{atm}$$

Option (A) is K_p at a total pressure of 1 atm; the data here are at 2 atm.

Answer: (C)

Common mistake: Using the vapour density 34.5 directly as the molar mass, or forgetting to multiply the mole fractions by the 2.0 atm total pressure.

Q.37 Thermodynamics – ΔH from ΔU and Gibbs energy • Standard • 120 s **Answer: B**

For the gas-phase reaction $2\,A(g) + B(g) \longrightarrow 2\,C(g)$ at 300 K, $\Delta U^\circ = -10.5$ kJ mol^{-1} and $\Delta S^\circ = -44.1$ J K^{-1} mol^{-1}. Treat all gases as ideal and take $R = 8.314$ J K^{-1} mol^{-1}. Under standard conditions at 300 K, ΔG° and the nature of the reaction are

(A) $+2.73$ kJ mol^{-1}; non-spontaneous
(B) $+0.24$ kJ mol^{-1}; non-spontaneous
(C) -26.2 kJ mol^{-1}; spontaneous
(D) $+5.22$ kJ mol^{-1}; non-spontaneous

Solution

Step 1 (ΔH from ΔU): $\Delta n_g = 2 - 3 = -1$.

$$\Delta H^\circ = \Delta U^\circ + \Delta n_g RT = -10.5 + (-1)(8.314 \times 300)/1000 = -12.99 \text{ kJ mol}^{-1}$$

Step 2 (Gibbs energy):

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = -12.99 - 300(-0.0441) = -12.99 + 13.23 = +0.24 \text{ kJ mol}^{-1}$$

$\Delta G^\circ > 0$, so the reaction is non-spontaneous under standard conditions at 300 K, though only just.

- (A) uses ΔU in place of ΔH.
- (C) reverses the sign of $T\Delta S$.
- (D) takes $\Delta n_g = +1$.

Answer: (B)

Common mistake: Putting ΔU straight into $\Delta G = \Delta H - T\Delta S$, or using $\Delta n_g = +1$; here the gas moles fall from 3 to 2.

Q.38 Coordination compounds – spectrochemical series and colour • Standard • 80 s **Answer: B**

The correct **increasing** order of the wavelength of light absorbed by the complexes $[Co(CN)_6]^{3-}$, $[Co(NH_3)_6]^{3+}$, $[Co(H_2O)_6]^{3+}$ and $[CoF_6]^{3-}$ is

(A) $[CoF_6]^{3-} < [Co(H_2O)_6]^{3+} < [Co(NH_3)_6]^{3+} < [Co(CN)_6]^{3-}$
(B) $[Co(CN)_6]^{3-} < [Co(NH_3)_6]^{3+} < [Co(H_2O)_6]^{3+} < [CoF_6]^{3-}$
(C) $[Co(NH_3)_6]^{3+} < [Co(CN)_6]^{3-} < [Co(H_2O)_6]^{3+} < [CoF_6]^{3-}$
(D) $[Co(CN)_6]^{3-} < [Co(H_2O)_6]^{3+} < [Co(NH_3)_6]^{3+} < [CoF_6]^{3-}$

Solution

All four complexes contain Co^{3+}, so the splitting Δ_o is set by the ligand. In the spectrochemical series:

$$F^- < H_2O < NH_3 < CN^-$$

The absorbed photon has energy $\Delta_o = hc/\lambda$. A larger Δ_o therefore means a **shorter** absorbed wavelength, and the wavelength increases as Δ_o decreases:

$$[Co(CN)_6]^{3-} < [Co(NH_3)_6]^{3+} < [Co(H_2O)_6]^{3+} < [CoF_6]^{3-}$$

Option (A) is the order of Δ_o itself, which is the reverse.

Answer: (B)

Common mistake: Taking the order of Δ_o as the order of wavelength, forgetting that $\lambda \propto 1/\Delta_o$.

Q.39 p-Block – groups 15, 16 and 17 (NCERT statements) • Standard • 110 s **Answer: D**

Consider the following statements.

I. The thermal stability of the hydrogen halides decreases in the order HF > HCl > HBr > HI.

II. The bond dissociation enthalpy of F_2 is greater than that of Cl_2.

III. H_3PO_3 is dibasic and acts as a reducing agent.

IV. Among the group 16 hydrides, H_2O has the highest boiling point but the lowest acidic character.

V. SF_6 is readily hydrolysed by water.

The correct statements are

(A) I, II and III only
(B) II, IV and V only
(C) III, IV and V only
(D) I, III and IV only

Solution

- **I: True.** The H–X bond enthalpy falls from HF to HI, so thermal stability falls in the same order.
- **II: False.** The NCERT order of bond dissociation enthalpy is $Cl_2 > Br_2 > F_2 > I_2$. The small F atoms suffer large lone-pair repulsion, which weakens the F–F bond.
- **III: True.** H_3PO_3, i.e. $HPO(OH)_2$, has two ionisable P–OH groups, so it is dibasic. Its P–H bond gives it reducing properties.
- **IV: True.** Hydrogen bonding gives H_2O the highest boiling point. Acidic character increases from H_2O to H_2Te as the H–E bond enthalpy decreases.
- **V: False.** SF_6 is exceptionally stable and unreactive, because the six F atoms sterically protect sulphur.

The correct statements are I, III and IV.

Answer: (D)

Common mistake: Assuming bond dissociation enthalpy falls regularly down the group and so judging the F–F bond stronger than the Cl–Cl bond.

Q.40 Aldehydes and ketones – structure from tests • Standard • 100 s **Answer: A**

A compound X, $C_5H_{10}O$, gives an orange precipitate with 2,4-DNP reagent and a yellow precipitate with I_2/NaOH, but does not reduce Tollens' reagent. Wolff–Kishner reduction of X gives 2-methylbutane. X is

(A) 3-Methylbutan-2-one (B) Pentan-2-one

(C) Pentan-3-one (D) 3-Methylbutanal

Solution

- **2,4-DNP positive:** X contains a C=O group.
- **No Tollens' reaction:** X is a ketone, not an aldehyde. This rules out (D).
- **Iodoform positive:** X is a methyl ketone, CH_3CO-R. This rules out pentan-3-one, (C).
- **Wolff–Kishner** turns C=O into CH_2 without changing the skeleton. Pentan-2-one would give n-pentane, while 3-methylbutan-2-one, $CH_3COCH(CH_3)_2$, gives $CH_3CH_2CH(CH_3)_2$, which is 2-methylbutane.

Answer: (A)

Common mistake: Choosing pentan-2-one from the iodoform test alone and ignoring the branched skeleton required by the Wolff–Kishner product.

Q.41 Practical chemistry – qualitative salt analysis • Standard • 110 s **Answer: C**

A solid salt X shows the following behaviour.

(i) With dilute H_2SO_4 it gives brisk effervescence. The gas is colourless and odourless, turns lime water milky, and does **not** turn acidified potassium dichromate paper green.

(ii) A solution of X in dilute HCl gives no precipitate when H_2S is passed through it.

(iii) After NH_4Cl and excess NH_4OH are added to this solution, no precipitate forms. When H_2S is then passed through the ammoniacal solution, a **white** precipitate appears.

X is

(A) $ZnSO_3$ (B) $MnCO_3$ (C) $ZnCO_3$ (D) $BaCO_3$

Solution

- **(i) Anion:** CO_2 is odourless, turns lime water milky and does not reduce dichromate. SO_2 from a sulphite has a suffocating smell and turns dichromate paper green. So the anion is carbonate, which rules out $ZnSO_3$.
- **(ii) and (iii) Cation:** there is no group II sulphide in acid and no group III hydroxide with NH_4Cl/NH_4OH. A sulphide precipitate in ammoniacal medium means **group IV**. ZnS is white, whereas MnS is buff (flesh-coloured), which rules out $MnCO_3$.
- Ba^{2+} (group V) gives no precipitate with H_2S even in ammoniacal medium, which rules out $BaCO_3$.

So X is $ZnCO_3$.

Answer: (C)

Common mistake: Forgetting that MnS is buff rather than white, or that SO_2 also turns lime water milky but is identified by dichromate.

Q.42 d-Block – oxidation states, electrode potentials and atomisation enthalpy (NCERT statements) • Standard • 100 s **Answer: A**

Consider the following statements about the 3d transition series.

I. Manganese shows the largest number of oxidation states, from +2 to +7.

II. Cu^{+} is stable in aqueous solution because it has a completely filled $3d^{10}$ configuration.

III. $E^{\circ}(Mn^{3+}/Mn^{2+})$ is much more positive than $E^{\circ}(Fe^{3+}/Fe^{2+})$, because of the extra stability of the d^5 ion Mn^{2+}.

IV. Zinc has the highest enthalpy of atomisation in the series, because its d orbitals are completely filled.

The correct statements are

(A) I and III only
(B) I, II and III only
(C) III and IV only
(D) I, III and IV only

Solution

- **I: True.** Mn $(3d^5 4s^2)$ can use all seven of its $3d$ and $4s$ electrons, and it shows every oxidation state from +2 to +7. This is the largest number in the 3d series.
- **II: False.** Cu^{+} disproportionates in water: $2\,Cu^{+}(aq) \longrightarrow Cu^{2+}(aq) + Cu\,(s)$. The much more negative hydration enthalpy of the small, highly charged Cu^{2+} more than compensates for the second ionisation enthalpy of Cu (NCERT).
- **III: True.** NCERT explains the high value for Mn^{3+}/Mn^{2+} (+1.57 V) by the particular stability of Mn^{2+} (d^5). The low value for Fe^{3+}/Fe^{2+} (+0.77 V) reflects the extra stability of Fe^{3+}, which is also d^5.
- **IV: False.** Enthalpy of atomisation depends on the number of unpaired electrons available for metallic bonding. It peaks in the middle of the series, around V and Cr. Zinc, with no unpaired d electrons, has the **lowest** value in the series.

The correct statements are I and III.

Answer: (A)

Common mistake: Treating the stable-looking d^{10} configuration as making $Cu^{+}(aq)$ stable, and ignoring the large hydration enthalpy of Cu^{2+}.

Q.43 Chemical kinetics – order from half-life and Arrhenius equation • Challenging • 200 s **Answer: B**

For the reaction A $\rightarrow$ products at 300 K, the half-life is 200 s when $[A]_0 = 0.10$ M and 400 s when $[A]_0 = 0.050$ M. At 320 K, starting with $[A]_0 = 0.20$ M, the concentration of A falls to 0.050 M in 75 s. The activation energy of the reaction is closest to ($R = 8.314\ J\,K^{-1}\,mol^{-1}$, $\ln 2 = 0.693$)

(A) 27.7 kJ mol^{-1}
(B) 55.3 kJ mol^{-1}
(C) 110.6 kJ mol^{-1}
(D) 66.8 kJ mol^{-1}

Solution

Step 1 (order): $t_{1/2} \propto [A]_0^{1-n}$. Halving $[A]_0$ doubles $t_{1/2}$, so $1-n=-1$ and $n=2$.

Step 2 (k at 300 K): for a second-order reaction $t_{1/2} = \dfrac{1}{k[A]_0}$:

$$k_{300} = \frac{1}{200 \times 0.10} = 0.050\ M^{-1}\,s^{-1}$$

Step 3 (k at 320 K):

$$\frac{1}{[A]} - \frac{1}{[A]_0} = kt \;\Rightarrow\; 20 - 5 = k_{320}(75) \;\Rightarrow\; k_{320} = 0.20\ M^{-1}\,s^{-1}$$

Step 4 (Arrhenius): $k_{320}/k_{300} = 4$.

$$E_a = \frac{R \ln 4\, T_1 T_2}{T_2 - T_1} = \frac{8.314 \times 1.386 \times 300 \times 320}{20} = 5.53 \times 10^4\ J\,mol^{-1} \approx 55.3\ kJ\,mol^{-1}$$

Option (D) comes from treating the reaction as first order: $t_{1/2}(320) = 37.5$ s against 200 s.

Answer: (B)

Common mistake: Assuming first order and reading '0.20 to 0.050 M' as two half-lives; for a second-order reaction successive half-lives are not equal.

Q.44 Multi-step organic conversion – aromatic substitution and diazonium chemistry • Challenging • 150 s **Answer: D**

Consider the following sequence.

Step 1: Toluene $\xrightarrow{\text{(i) } KMnO_4/KOH,\ \Delta;\ \text{(ii) } H_3O^+}$ A

Step 2: A $\xrightarrow{\text{conc. } HNO_3/\text{conc. } H_2SO_4,\ \Delta}$ B

Step 3: B $\xrightarrow{\text{(i) Sn/HCl; (ii) neutralise}}$ C

Step 4: C $\xrightarrow{NaNO_2/HCl,\ 0\text{–}5\ ^\circ C}$ D $\xrightarrow{H_2O,\ \text{warm}}$ E

The major product E is

(A) 2-Hydroxybenzoic acid
(B) 3-Chlorobenzoic acid
(C) 4-Hydroxybenzoic acid
(D) 3-Hydroxybenzoic acid

Solution

- **A:** side-chain oxidation turns $C_6H_5CH_3$ into benzoic acid, C_6H_5COOH.
- **B:** $-COOH$ is a deactivating **meta**-director, so nitration gives 3-nitrobenzoic acid. (Had toluene been nitrated first, the products would be ortho/para.)
- **C:** Sn/HCl reduces $-NO_2$ to $-NH_2$, giving 3-aminobenzoic acid.
- **D:** diazotisation at 0–5 °C gives the arenediazonium salt, 3-carboxybenzenediazonium chloride.
- **E:** warming the diazonium salt in water replaces $-N_2^+$ by $-OH$. The product is 3-hydroxybenzoic acid. (A Sandmeyer reaction would need CuCl to give the chloro compound.)

Answer: (D)

Common mistake: Treating $-COOH$ as an ortho/para director, as for the original $-CH_3$, and answering salicylic or 4-hydroxybenzoic acid.

Q.45 Electrochemistry – Kohlrausch's law, degree of dissociation and pH • Challenging • 200 s **Answer: C**

At 298 K the limiting molar conductivities of HCl, NaCl and CH_3COONa are 425.9, 126.4 and 91.0 $S\,cm^2\,mol^{-1}$ respectively. A 0.010 M solution of acetic acid has conductivity 1.60×10^{-4} $S\,cm^{-1}$ at 298 K. The pH of this acetic acid solution is closest to (log 4.1 = 0.613)

(A) 2.39 (B) 4.39 (C) 3.39 (D) 3.60

Solution

Step 1 (Kohlrausch):

$$\Lambda_m^\circ(CH_3COOH) = \Lambda^\circ(HCl) + \Lambda^\circ(CH_3COONa) - \Lambda^\circ(NaCl) = 425.9 + 91.0 - 126.4 = 390.5\ S\,cm^2\,mol^{-1}$$

Step 2 (molar conductivity):

$$\Lambda_m = \frac{1000\,\kappa}{c} = \frac{1000 \times 1.60 \times 10^{-4}}{0.010} = 16.0\ S\,cm^2\,mol^{-1}$$

Step 3 (α and pH):

$$\alpha = \frac{16.0}{390.5} = 0.0410, \qquad [H^+] = c\alpha = 4.1 \times 10^{-4}\ M$$

$$pH = 4 - \log 4.1 = 4 - 0.613 = 3.39$$

Option (D) results from adding all three limiting conductivities (643.3), which gives pH $\approx$ 3.60.

Answer: (C)

Common mistake: Combining the limiting conductivities wrongly (adding NaCl instead of subtracting it), or omitting the factor 1000 in $\Lambda_m = 1000\kappa/c$.

Q.46 Coordination compounds – geometrical and optical isomer count • Standard • 90 s **Answer: 9**

Find the total number of stereoisomers (geometrical and optical counted together) summed over the following four complexes: $[Cr(C_2O_4)_2Cl_2]^{3-}$, $[Co(NH_3)_3Cl_3]$, $[Co(en)_3]^{3+}$ and $[Co(NH_3)_4Cl_2]^+$. ($C_2O_4^{2-}$ and en are symmetrical bidentate ligands.)

Solution

All four complexes are octahedral.

- $\mathbf{[Cr(C_2O_4)_2Cl_2]^{3-}}$, type $[M(AA)_2X_2]$: the trans isomer has a mirror plane and is optically inactive. The cis isomer is chiral and exists as a pair of enantiomers. Total $= 1 + 2 = 3$.

- $\mathbf{[Co(NH_3)_3Cl_3]}$, type $[MA_3B_3]$: there are fac and mer isomers, and both have mirror planes. Total $= 2$.
- $\mathbf{[Co(en)_3]^{3+}}$, type $[M(AA)_3]$: there is no geometrical isomer, but the complex is chiral and exists as the d and l forms. Total $= 2$.
- $\mathbf{[Co(NH_3)_4Cl_2]^{+}}$, type $[MA_4B_2]$: there are cis and trans isomers, and both are optically inactive. Total $= 2$.

Sum $= 3 + 2 + 2 + 2 = 9$.

Answer: 9

Common mistake: Counting only 2 isomers for $[Cr(C_2O_4)_2Cl_2]^{3-}$ (cis and trans) and missing that the cis form is chiral, or counting 1 for $[Co(en)_3]^{3+}$.

Q.47

Chemical bonding – VSEPR and square planar species • Foundation • 75 s **Answer: 5**

How many of the following species have a **square planar** geometry? XeF_4, $[Ni(CN)_4]^{2-}$, SF_4, $[PtCl_4]^{2-}$, BrF_4^-, ICl_4^-, $[NiCl_4]^{2-}$, CCl_4, XeO_4, $[Zn(NH_3)_4]^{2+}$

Solution

- XeF_4: 4 bond pairs + 2 lone pairs (sp^3d^2), square planar ✓.
- $[Ni(CN)_4]^{2-}$: Ni^{2+} (d^8) with strong-field CN^-, dsp^2, square planar ✓.
- SF_4: 4 bond pairs + 1 lone pair, see-saw.
- $[PtCl_4]^{2-}$: Pt^{2+} ($5d^8$), dsp^2, square planar ✓.
- BrF_4^- and ICl_4^-: 4 bond pairs + 2 lone pairs on the halogen, square planar ✓✓.
- $[NiCl_4]^{2-}$: weak-field Cl^-, sp^3, tetrahedral.
- CCl_4 and XeO_4: tetrahedral.
- $[Zn(NH_3)_4]^{2+}$: d^{10}, sp^3, tetrahedral.

Count $= 5$.

Answer: 5

Common mistake: Counting $[NiCl_4]^{2-}$ as square planar like $[Ni(CN)_4]^{2-}$; the weak-field Cl^- ligand gives a tetrahedral sp^3 complex.

Q.48

Ionic equilibrium – neutralisation and salt hydrolysis • Standard • 100 s **Answer: 9**

At 298 K, 50.0 mL of 0.20 M acetic acid is mixed with 50.0 mL of 0.20 M NaOH. Given $K_a(CH_3COOH) = 1.0 \times 10^{-5}$ and $K_w = 1.0 \times 10^{-14}$, find the pH of the resulting solution.

Solution

Neutralisation: 0.010 mol acid and 0.010 mol NaOH react exactly, leaving 0.010 mol CH_3COONa in 100 mL, so $c = 0.10$ M.

Hydrolysis of the acetate ion: $CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + OH^-$, with

$$K_h = \frac{K_w}{K_a} = 10^{-9}, \qquad [OH^-] = \sqrt{K_h c} = \sqrt{10^{-9} \times 0.10} = 10^{-5}\ \text{M}$$

So pOH $= 5$ and pH $= 9$. Equivalently, $\text{pH} = 7 + \frac{1}{2}pK_a + \frac{1}{2}\log c = 7 + 2.5 - 0.5 = 9$.

Answer: 9

Common mistake: Using $c = 0.20$ M and ignoring the dilution to 100 mL on mixing, which gives about 9.15.

Q.49

Solutions – association and depression in freezing point • Standard • 120 s **Answer: 80**

Benzoic acid dimerises in benzene. When 1.22 g of benzoic acid is dissolved in 40.0 g of benzene, the freezing point is lowered by 0.768 K. Given K_f(benzene) $= 5.12$ K kg mol^{-1} and molar mass of benzoic acid $= 122$ g mol^{-1}, find the percentage of benzoic acid molecules that are associated as dimers.

Solution

Molality, ignoring association:

$$m = \frac{1.22/122}{0.0400} = 0.250\ \text{mol kg}^{-1}$$

$$i = \frac{\Delta T_f}{K_f m} = \frac{0.768}{5.12 \times 0.250} = 0.600$$

For $2\,A \rightleftharpoons A_2$ with degree of association β:

$$i = 1 - \frac{\beta}{2} = 0.600 \ \Rightarrow\ \beta = 0.80$$

So **80%** of the molecules are associated.

Answer: 80

Common mistake: Writing $i = 1 - \beta$ (as if association halved nothing), which gives 40%; for dimerisation $i = 1 - \beta/2$.

Q.50

Multi-step organic conversion – aldol, Grignard, dehydration, ozonolysis • Challenging • 180 s **Answer: 58**

Consider the following sequence.

Step 1: $CH_3CHO \xrightarrow{\text{(i) dil. NaOH; (ii) heat}} A \xrightarrow{H_2\ (\text{excess})/Ni} B$

Step 2: $B \xrightarrow{PCC} C \xrightarrow{\text{(i) } CH_3MgBr;\ \text{(ii) } H_3O^+} D$

Step 3: $D \xrightarrow{\text{conc. } H_2SO_4,\ 443\ K} E\ (\text{major}) \xrightarrow{\text{(i) } O_3;\ \text{(ii) } Zn/H_2O} F + G$

F and G are different carbonyl compounds. Find the molar mass (in $g\,mol^{-1}$) of the **heavier** of F and G. (C = 12, H = 1, O = 16)

Solution

- **A:** aldol condensation gives but-2-enal, $CH_3CH{=}CHCHO$.
- **B:** excess H_2/Ni reduces both the C=C and the CHO group, giving butan-1-ol, $CH_3CH_2CH_2CH_2OH$.
- **C:** PCC oxidises a primary alcohol only as far as the aldehyde, giving butanal, $CH_3CH_2CH_2CHO$.
- **D:** CH_3MgBr adds to the aldehyde, giving pentan-2-ol, $CH_3CH_2CH_2CH(OH)CH_3$.
- **E:** acid-catalysed dehydration follows Saytzeff's rule, giving pent-2-ene, $CH_3CH_2CH{=}CHCH_3$, as the major product.
- **F + G:** ozonolysis cleaves the C2=C3 bond into ethanal CH_3CHO ($M = 44$) and propanal CH_3CH_2CHO ($M = 58$).

The heavier product is propanal, $M = 3(12) + 6(1) + 16 = 58$.

Answer: 58

Common mistake: Taking the terminal alkene pent-1-ene as the major dehydration product, which leads to butanal (72) and HCHO instead of propanal (58).

PART C : MATHEMATICS — Detailed solutions

Q.51

Sets and relations – equivalence closure • Foundation • 75 s **Answer: A**

Let $A = \{1, 2, 3, 4, 5, 6\}$ and let R be the smallest equivalence relation on A that contains the ordered pairs $(1, 2)$, $(2, 3)$ and $(4, 5)$. The number of ordered pairs in R is

(A) 14 (B) 13 (C) 11 (D) 9

Solution

An equivalence relation is the same as a partition of A into classes, with $(a, b) \in R$ exactly when a, b lie in the same class.

$(1, 2), (2, 3) \in R$ force $1, 2, 3$ into one class (transitivity). $(4, 5) \in R$ puts $4, 5$ together. Nothing links 6 to anything, and the *smallest* relation adds no further links. So the classes are

$$\{1, 2, 3\}, \qquad \{4, 5\}, \qquad \{6\}.$$

A class with k elements contributes k^2 ordered pairs (every (a, b) inside it, including (a, a)):

$$|R| = 3^2 + 2^2 + 1^2 = 9 + 4 + 1 = 14.$$

Answer: (A)

Common mistake: Forgetting the reflexive pair $(6, 6)$ gives 13; forgetting $(1, 3), (3, 1)$ from transitivity gives 12 or less.

Q.52

Complex numbers – modulus equation • Foundation • 80 s **Answer: C**

Let z be a complex number such that $|z| + iz = 1 + 3i$. Then z is equal to

(A) $3 - 4i$ (B) $4 + 3i$ (C) $3 + 4i$ (D) $-3 + 4i$

Solution

Put $z = x + iy$. Then $iz = ix + i^2y = -y + ix$, so

$$|z| + iz = \left(\sqrt{x^2 + y^2} - y\right) + ix = 1 + 3i.$$

Comparing imaginary parts: $x = 3$.

Comparing real parts:

$$\sqrt{9 + y^2} = 1 + y \;\Rightarrow\; 9 + y^2 = 1 + 2y + y^2 \;\Rightarrow\; y = 4.$$

(This is valid since $1 + y = 5 > 0$.)

Hence $z = 3 + 4i$, with $|z| = 5$.

Check: $|z| + iz = 5 + (3i - 4) = 1 + 3i$. By contrast, $-3 + 4i$ gives $1 - 3i$, $3 - 4i$ gives $9 + 3i$ and $4 + 3i$ gives $2 + 4i$.

Answer: (C)

Common mistake: Taking $i \cdot iy = +y$ (forgetting $i^2 = -1$), which leads to $3 - 4i$; or matching the real part of iz with x instead of $-y$.

Q.53

Statistics – corrected mean and variance • Foundation • 90 s **Answer: B**

The mean and the variance of 10 observations are 12 and 16 respectively. Later it was found that one observation had been wrongly recorded as 18 instead of the correct value 8. The correct variance is

(A) 16 (B) 13 (C) 12 (D) 15

Solution

From the given data:

$$\sum x = 10 \times 12 = 120, \qquad \frac{\sum x^2}{10} - 12^2 = 16 \;\Rightarrow\; \sum x^2 = 10(16 + 144) = 1600.$$

Correct the totals by removing 18 and inserting 8:

$$\sum x = 120 - 18 + 8 = 110, \qquad \sum x^2 = 1600 - 324 + 64 = 1340.$$

Correct mean $= 11$, so the correct variance is

$$\frac{1340}{10} - 11^2 = 134 - 121 = 13.$$

Answer: (B)

Common mistake: Correcting $\sum x^2$ but still subtracting the old mean squared 144, or correcting only the mean.

Q.54 Straight lines – image of a point and distance • Foundation • 90 s **Answer: D**

Q is the image of the point $P(4,1)$ in the line $x+2y=1$. The perpendicular distance of Q from the line $3x-4y=3$ is

(A) 1 (B) $\frac{13}{5}$

(C) $\frac{9}{5}$ (D) 3

Solution

Image of (x_0,y_0) in $ax+by+c=0$:

$$\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{-2(ax_0+by_0+c)}{a^2+b^2}.$$

For $x+2y-1=0$ and $P(4,1)$: $ax_0+by_0+c=4+2-1=5$. So the common ratio is $\frac{-2(5)}{5}=-2$, which gives

$$Q=(4-2,\ 1-4)=(2,-3).$$

Distance of Q from $3x-4y-3=0$:

$$\frac{|3(2)-4(-3)-3|}{5}=\frac{|6+12-3|}{5}=\frac{15}{5}=3.$$

Answer: (D)

Common mistake: Using the foot of the perpendicular $(3,-1)$ instead of the image, or the distance of P itself $(=1)$.

Q.55 Binomial theorem – coefficient in a product • Foundation • 100 s **Answer: D**

The coefficient of x^3 in the expansion of $(1+x^3)\left(x^2-\frac{2}{x}\right)^6$ is

(A) -160 (B) 240 (C) 400 (D) 80

Solution

General term of $\left(x^2-\frac{2}{x}\right)^6$:

$$T_{r+1}=\binom{6}{r}(x^2)^{6-r}\left(-\frac{2}{x}\right)^r=\binom{6}{r}(-2)^r\,x^{12-3r}.$$

The coefficient of x^3 in the product is (coefficient of x^3) + (coefficient of x^0) in $\left(x^2-\frac{2}{x}\right)^6$.

- x^3: $12-3r=3\Rightarrow r=3$: $\binom{6}{3}(-2)^3=20(-8)=-160$.
- x^0: $12-3r=0\Rightarrow r=4$: $\binom{6}{4}(-2)^4=15(16)=240$.

Required coefficient $=-160+240=80$.

Answer: (D)

Common mistake: Taking only the x^3 term from the binomial (-160) and forgetting that $x^3\cdot x^0$ also contributes.

Q.56 3D geometry – distance measured along a line • Foundation • 100 s **Answer: B**

The distance of the point $P(4,-1,5)$ from the plane $x-y+z=5$, measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$, is

(A) $\frac{5}{\sqrt{3}}$ (B) 5

(C) 7 (D) $\frac{5}{7}$

Solution

The line has direction ratios $(2,3,-6)$, of length $\sqrt{4+9+36}=7$, so the unit vector is $\frac{1}{7}(2,3,-6)$.
A point at distance r from P along this direction is

$$\left(4+\frac{2r}{7},\ -1+\frac{3r}{7},\ 5-\frac{6r}{7}\right).$$

Put it in $x-y+z=5$:

$$4+\frac{2r}{7}+1-\frac{3r}{7}+5-\frac{6r}{7}=5\ \Rightarrow\ 10-r=5\ \Rightarrow\ r=5.$$

So the required distance is 5. (The perpendicular distance, $5/\sqrt{3}$, is a different quantity.)

Answer: (B)

Common mistake: Giving the perpendicular distance $5/\sqrt{3}$, or using direction ratios without normalising (getting $5/7$).

Q.57 Sequences and series – nth term from S(n), telescoping sum • Standard • 110 s **Answer: A**

For a sequence $\{a_n\}$, the sum of the first n terms is $S_n = n^2 + 3n$ for every $n \in \mathbb{N}$. Then $\displaystyle\sum_{k=1}^{20} \frac{1}{a_k\, a_{k+1}}$ is equal to

(A) $\frac{5}{44}$ (B) $\frac{1}{11}$

(C) $\frac{21}{88}$ (D) $\frac{5}{22}$

Solution

For $n \geq 2$: $a_n = S_n - S_{n-1} = (n^2+3n) - ((n-1)^2 + 3(n-1)) = 2n+2$. Also $a_1 = S_1 = 4 = 2(1)+2$. So $a_n = 2n+2$ for all n.

Then

$$\frac{1}{a_k a_{k+1}} = \frac{1}{(2k+2)(2k+4)} = \frac{1}{4}\cdot\frac{1}{(k+1)(k+2)} = \frac{1}{4}\left(\frac{1}{k+1} - \frac{1}{k+2}\right).$$

Telescoping from $k = 1$ to 20:

$$\sum_{k=1}^{20} \frac{1}{a_k a_{k+1}} = \frac{1}{4}\left(\frac{1}{2} - \frac{1}{22}\right) = \frac{1}{4}\cdot\frac{10}{22} = \frac{5}{44}.$$

Answer: (A)

Common mistake: Forgetting the factor $\frac{1}{4}$ (getting 5/11-type values) or telescoping up to $\frac{1}{21}$ instead of $\frac{1}{22}$.

Q.58 Probability – conditional probability with counting • Standard • 110 s **Answer: C**

Two distinct numbers are chosen at random from $\{1, 2, 3, \ldots, 20\}$. Given that their sum is even, the probability that both chosen numbers are prime is

(A) $\frac{21}{190}$ (B) $\frac{14}{45}$

(C) $\frac{7}{30}$ (D) $\frac{7}{19}$

Solution

The sum is even exactly when both numbers are even or both are odd. There are 10 even and 10 odd numbers, so

$$n(\text{sum even}) = \binom{10}{2} + \binom{10}{2} = 45 + 45 = 90.$$

Primes up to 20: $2, 3, 5, 7, 11, 13, 17, 19$. If 2 is chosen with an odd prime the sum is odd, and there is only one even prime. So both primes must be odd, from $\{3, 5, 7, 11, 13, 17, 19\}$ (7 numbers):

$$n(\text{both prime and sum even}) = \binom{7}{2} = 21.$$

$$P = \frac{21}{90} = \frac{7}{30}.$$

Answer: (C)

Common mistake: Including the prime 2 (giving $\binom{8}{2}/90 = 14/45$), or dividing by $\binom{20}{2} = 190$ instead of conditioning.

Q.59 Quadratic equations – location of roots • Standard • 140 s **Answer: B**

The set of all real values of k for which both roots of $x^2 - 2kx + k^2 + k - 5 = 0$ are real and lie in the open interval $(-1, 5)$ is

(A) $(1, 5]$ (B) $(1, 4)$ (C) $(-4, 4)$ (D) $(1, 5)$

Solution

Let $f(x) = x^2 - 2kx + k^2 + k - 5$. Both roots lie in $(-1, 5)$ if and only if all four conditions hold.

1. **Real roots:** $D = 4k^2 - 4(k^2 + k - 5) = 4(5 - k) \geq 0 \Rightarrow k \leq 5$.

2. $f(-1) > 0$: $1 + 2k + k^2 + k - 5 = k^2 + 3k - 4 = (k+4)(k-1) > 0 \Rightarrow k < -4$ or $k > 1$.
3. $f(5) > 0$: $25 - 10k + k^2 + k - 5 = k^2 - 9k + 20 = (k-4)(k-5) > 0 \Rightarrow k < 4$ or $k > 5$.
4. **Vertex inside:** $-1 < k < 5$.

Intersecting these gives $1 < k < 4$.

Check the end points. At $k = 4$ a root equals 5 (excluded). At $k = 1$ a root equals -1 (excluded).

Answer: (B)

Common mistake: Dropping the condition $f(5) > 0$ (or the vertex condition), which leaves $(1, 5]$.

Q.60

Indefinite integration – exponential times (g + g') form • Standard • 120 s **Answer: A**

Let $F(x) = \int e^x \dfrac{x^2+3x+3}{(x+2)^2}\,dx$ for $x > -2$, with $F(0) = \dfrac{1}{2}$. Then $F(1)$ is equal to

(A) $\dfrac{2e}{3}$ (B) $\dfrac{3e}{4}$

(C) $\dfrac{2e}{3} - \dfrac{1}{2}$ (D) $\dfrac{e}{3}$

Solution

Look for g with $g + g' = \dfrac{x^2+3x+3}{(x+2)^2}$. Try $g(x) = \dfrac{x+1}{x+2} = 1 - \dfrac{1}{x+2}$. Then $g'(x) = \dfrac{1}{(x+2)^2}$ and

$$g + g' = \frac{(x+1)(x+2)+1}{(x+2)^2} = \frac{x^2+3x+3}{(x+2)^2}.$$

Since $\int e^x(g+g')\,dx = e^x g(x) + C$:

$$F(x) = e^x \frac{x+1}{x+2} + C.$$

$F(0) = \frac{1}{2} + C = \frac{1}{2} \Rightarrow C = 0$. So

$$F(1) = e \cdot \frac{2}{3} = \frac{2e}{3}.$$

Answer: (A)

Common mistake: Choosing $g = \dfrac{1}{x+2}$-type guesses, or keeping a nonzero constant ($\frac{2e}{3} - \frac{1}{2}$) by mis-evaluating $F(0)$.

Q.61

Inverse trigonometric functions – principal value branches • Standard • 120 s **Answer: D**

Considering principal values, the set of all real x satisfying $2\tan^{-1} x + \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = \pi$ is

(A) $\{1\}$ (B) $(-\infty, -1] \cup [1, \infty)$ (C) $(1, \infty)$ (D) $[1, \infty)$

Solution

Let $x = \tan\theta$ with $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$, so $\dfrac{2x}{1+x^2} = \sin 2\theta$ and $2\theta \in (-\pi, \pi)$.

$$\sin^{-1}(\sin 2\theta) = \begin{cases} 2\theta, & |x| \le 1 \ (|2\theta| \le \frac{\pi}{2}) \\ \pi - 2\theta, & x > 1 \\ -\pi - 2\theta, & x < -1 \end{cases}$$

So the left side $= 2\theta + \sin^{-1}(\sin 2\theta)$ equals:

- $4\tan^{-1} x$ for $|x| \le 1$. This equals π only at $x = 1$.
- π for every $x > 1$.
- $-\pi$ for every $x < -1$.

So the solution set is $\{1\} \cup (1, \infty) = [1, \infty)$.

Answer: (D)

Common mistake: Using $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1} x$ for all x (getting only $x = 1$), or forgetting the endpoint $x = 1$.

Q.62 Circles – director circle, points of contact and the circle on CP as diameter • Standard • 120 s **Answer: C**

A point P on the line $x+y=6$ is such that the two tangents drawn from P to the circle $x^2+y^2-2x-2y-2=0$ are perpendicular to each other. The equation of the circle passing through P and the two points of contact of these tangents is

(A) $x^2+y^2-4x-4y+4=0$ (B) $x^2+y^2-2x-2y-6=0$

(C) $x^2+y^2-4x-4y+6=0$ (D) $x^2+y^2-6x-6y+16=0$

Solution

The given circle has centre $C(1,1)$ and radius $r=\sqrt{1+1+2}=2$.

Finding P. Perpendicular tangents are drawn only from points of the director circle, which is centred at C with radius $r\sqrt{2}=2\sqrt{2}$. The distance of C from $x+y=6$ is $\dfrac{|1+1-6|}{\sqrt{2}}=2\sqrt{2}$. So the line touches the director circle at exactly one point, the foot of the perpendicular from C: Points on the normal through C are $(1+t,\,1+t)$. Putting one in $x+y=6$ gives $2+2t=6$, so $t=2$ and $P=(3,3)$. (Check: $CP=\sqrt{4+4}=2\sqrt{2}$.)

Required circle. If A and B are the points of contact, then $\angle CAP=\angle CBP=90^\circ$. So C, A, P, B lie on the circle with CP as diameter:

$$(x-1)(x-3)+(y-1)(y-3)=0 \;\Rightarrow\; x^2+y^2-4x-4y+6=0.$$

Check: the chord of contact of P is $x+y=4$, which meets the given circle at $A(3,1)$ and $B(1,3)$. Both points, and $P(3,3)$, satisfy $x^2+y^2-4x-4y+6=0$.

Answer: (C)

Common mistake: Taking the director circle itself $(x^2+y^2-2x-2y-6=0)$, or using radius 2 (that of the given circle) for the circle on CP as diameter.

Q.63 Vector algebra – vector equations (cross and dot products given) • Standard • 150 s **Answer: C**

Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{k}$. A vector $\vec{c}$ satisfies $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=4$. If θ is the angle between $\vec{a}$ and $\vec{c}$, then

(A) $\sin\theta=\dfrac{2\sqrt{2}}{3}$ (B) $\cos\theta=\dfrac{1}{3}$

(C) $\sin\theta=\dfrac{1}{3}$ (D) $\tan\theta=2\sqrt{2}$

Solution

Note that $\vec{a}\cdot\vec{b}=1-1=0$, so the system is consistent. Cross $\vec{a}$ with $\vec{a}\times\vec{c}=\vec{b}$:

$$\vec{a}\times(\vec{a}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{a}-|\vec{a}|^2\vec{c}=\vec{a}\times\vec{b}.$$

Here $|\vec{a}|^2=6$ and $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&1\\1&0&-1\end{vmatrix}=-2\hat{i}+2\hat{j}-2\hat{k}$. So

$$\vec{c}=\frac{4\vec{a}-\vec{a}\times\vec{b}}{6}=\frac{(4,8,4)-(-2,2,-2)}{6}=(1,1,1).$$

Check: $\vec{a}\times\vec{c}=(1,0,-1)=\vec{b}$ and $\vec{a}\cdot\vec{c}=4$.

Now $|\vec{a}|=\sqrt{6}$ and $|\vec{c}|=\sqrt{3}$:

$$\cos\theta=\frac{4}{\sqrt{18}}=\frac{2\sqrt{2}}{3},\qquad \sin\theta=\frac{|\vec{a}\times\vec{c}|}{|\vec{a}||\vec{c}|}=\frac{\sqrt{2}}{\sqrt{18}}=\frac{1}{3}.$$

Answer: (C)

Common mistake: Sign slip in $\vec{c}=(k\vec{a}-\vec{a}\times\vec{b})/|\vec{a}|^2$ (using $+\vec{a}\times\vec{b}$), which gives a wrong $|\vec{c}|$; or swapping sin and cos.

Q.64 Limits – GIF sandwich and series expansions • Standard • 140 s **Answer: A**

Let $a=\lim\limits_{x\to0}x^2\left[\dfrac{3}{x^2}\right]$, where $[t]$ denotes the greatest integer less than or equal to t, and let

$$b=\lim_{x\to0}\frac{e^{2x^3}-\left(1-3x^3\right)^{1/3}+\left(\sqrt{1-x^2}-1\right)\sin 2x}{x\sin^2 x}.$$

Then $a+b$ is equal to

(A) 5 (B) 4 (C) 6 (D) 3

Solution

Finding a. Since $\frac{3}{x^2} - 1 < \left[\frac{3}{x^2}\right] \le \frac{3}{x^2}$, multiplying by $x^2 > 0$ gives

$$3 - x^2 < x^2 \left[\frac{3}{x^2}\right] \le 3.$$

By the sandwich theorem, $a = 3$.

Finding b. The denominator is $x \sin^2 x \sim x^3$. Expand the numerator up to x^3:

- $e^{2x^3} = 1 + 2x^3 + O(x^6)$
- $(1 - 3x^3)^{1/3} = 1 - x^3 + O(x^6)$
- $\left(\sqrt{1 - x^2} - 1\right) \sin 2x = \left(-\frac{x^2}{2} + \cdots\right)(2x + \cdots) = -x^3 + O(x^5)$

Numerator $= (1 + 2x^3) - (1 - x^3) - x^3 + \cdots = 2x^3 + \cdots$. So

$$b = \lim_{x \to 0} \frac{2x^3}{x^3} = 2.$$

Therefore $a + b = 3 + 2 = 5$.

Answer: (A)

Common mistake: Treating $[3/x^2]$ as giving 0 in the limit (getting $a = 0$), or expanding $(1 - 3x^3)^{1/3}$ as $1 - 3x^3$.

Q.65

Matrices and determinants – adjoint chains • Standard • 150 s **Answer: D**

Let $A = \begin{bmatrix} 4 & 1 \\ 5 & 2 \end{bmatrix}$ and, for $p \in \mathbb{N}$, let $B_p = A^p \underbrace{\operatorname{adj}(\operatorname{adj}(\cdots \operatorname{adj}}_{p \text{ times}}(A) \cdots))$. Which of the following statements is **incorrect**?

(A) $\operatorname{tr}(B_1) = 6$

(B) $\operatorname{adj}(\operatorname{adj}(B_2)) = A^3$

(C) $\det(B_2) = 27$

(D) $\sum_{p=1}^{\infty} \det(B_p^{-1}) = \frac{1}{3}$

Solution

Here $\det A = 8 - 5 = 3$. For any 2×2 matrix M: $\operatorname{adj}(\operatorname{adj} M) = M$ and $\det(\operatorname{adj} M) = \det M$.

So applying adj p times gives adj A when p is odd and A when p is even. In both cases the determinant is 3. Hence

$$\det B_p = (\det A)^p \cdot 3 = 3^{p+1}.$$

- **(A)** $B_1 = A$ adj $A = (\det A)I = 3I$, so tr $B_1 = 6$. Correct.
- **(C)** $\det B_2 = 3^3 = 27$. Correct.
- **(B)** $B_2 = A^2 \cdot A = A^3$ is 2×2, so $\operatorname{adj}(\operatorname{adj} B_2) = B_2 = A^3$. Correct.
- **(D)** $\det(B_p^{-1}) = 3^{-(p+1)}$, so

$$\sum_{p=1}^{\infty} 3^{-(p+1)} = \frac{1/9}{1 - 1/3} = \frac{1}{6} \neq \frac{1}{3}.$$

Incorrect.

Answer: (D)

Common mistake: Using the 3×3 rule $\det(\operatorname{adj} M) = (\det M)^2$ for a 2×2 matrix, or starting the GP at 3^{-1}.

Q.66

Hyperbola – equation from foci and a point; tangent • Standard • 150 s **Answer: B**

A hyperbola has its centre at the origin, its transverse axis along the x-axis and its foci at $(\pm 5, 0)$. It passes through the point $P\left(4\sqrt{2}, 3\right)$. The tangent to the hyperbola at P meets the x-axis at T and the y-axis at U. If O is the origin, the area of triangle OTU is

(A) $6\sqrt{2}$

(B) $3\sqrt{2}$

(C) $\frac{3\sqrt{2}}{2}$

(D) 12

Solution

Let the hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with $a^2+b^2=25$. Since P lies on it:

$$\frac{32}{a^2}-\frac{9}{25-a^2}=1 \;\Rightarrow\; 32(25-a^2)-9a^2=a^2(25-a^2) \;\Rightarrow\; a^4-66a^2+800=0.$$

So $a^2=16$ or 50. Since $a^2<25$, $a^2=16$ and $b^2=9$.

Tangent at P:

$$\frac{4\sqrt{2}\,x}{16}-\frac{3y}{9}=1 \;\Rightarrow\; \frac{x}{2\sqrt{2}}-\frac{y}{3}=1.$$

So $T=(2\sqrt{2},0)$ and $U=(0,-3)$.

$$\text{Area}=\frac{1}{2}\cdot 2\sqrt{2}\cdot 3=3\sqrt{2}.$$

(The value $12=ab$ is the area cut off by the tangent with the **asymptotes**, not the axes.)

Answer: (B)

Common mistake: Accepting $a^2=50$ (which makes $b^2<0$), or confusing this triangle with the asymptote triangle of area $ab=12$.

Q.67

Continuity and differentiability – max function and product • Standard • 170 s **Answer: D**

Let $f(x)=\max\{x^2-2,\ |x|\}$ and $g(x)=x\,f(x)$ for $x\in(-3,3)$. If m is the number of points in $(-3,3)$ at which f is not differentiable and n is the number of points in $(-3,3)$ at which g is not differentiable, then $m+n$ is equal to

(A) 4 (B) 3 (C) 6 (D) 5

Solution

$x^2-2=|x| \iff |x|^2-|x|-2=0 \iff |x|=2$. For $|x|<2$ we have $x^2-2<|x|$. So

$$f(x)=\begin{cases}|x|, & |x|\le 2\\ x^2-2, & 2<|x|<3.\end{cases}$$

Points where f is not differentiable:

- $x=0$: the corner of $|x|$.
- $x=2$: the left slope is 1 and the right slope is $2x=4$.
- $x=-2$: by symmetry.

So $m=3$.

Points where $g=xf$ is not differentiable:

- $x=0$: $g=x|x|$ near 0, and $g'(0)=\lim_{h\to 0}|h|=0$, so g is differentiable there.
- $x=2$: $g'(2^-)=f(2)+2\cdot 1=4$ and $g'(2^+)=f(2)+2\cdot 4=10$, which are unequal.
- $x=-2$: $g'(-2^+)=f(-2)+(-2)(-1)=4$ and $g'(-2^-)=f(-2)+(-2)(-4)=10$, which are unequal.

So $n=2$, and $m+n=5$.

Answer: (D)

Common mistake: Assuming multiplication by x removes every corner (getting $n=0$), or missing that the factor x removes the corner only at $x=0$, where x itself vanishes.

Q.68

Differential equations – linear DE, then optimisation • Challenging • 180 s **Answer: C**

Let $y=y(x)$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, be the solution of $\frac{dy}{dx}+2y\tan x=\sin x$ with $y\left(\frac{\pi}{4}\right)=0$. Then the maximum value of $y(x)$ on $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is

(A) $\frac{\sqrt{2}}{4}$ (B) $\frac{1}{8}$

(C) $\frac{\sqrt{2}}{8}$ (D) 0

Solution

This is a linear DE. The integrating factor is $e^{\int 2\tan x\,dx}=e^{2\ln\sec x}=\sec^2 x$ (here $\cos x>0$). So

$$\frac{d}{dx}\left(y\sec^2 x\right)=\sin x\sec^2 x=\sec x\tan x \;\Rightarrow\; y\sec^2 x=\sec x+C.$$

$y(\pi/4) = 0$ gives $0 = \sqrt{2} + C$, so $C = -\sqrt{2}$. Then

$$y = \cos x - \sqrt{2}\cos^2 x.$$

Put $c = \cos x \in (0, 1]$: $y = c - \sqrt{2}\,c^2$. This is a downward parabola in c with vertex at $c = \dfrac{1}{2\sqrt{2}} \in (0, 1]$. So

$$y_{\max} = \frac{1}{2\sqrt{2}} - \sqrt{2}\cdot\frac{1}{8} = \frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8},$$

attained at $x = \pm\cos^{-1}\dfrac{1}{2\sqrt{2}}$.

Answer: (C)

Common mistake: Putting the vertex of $c - \sqrt{2}c^2$ at $c = \frac{1}{\sqrt{2}}$ (using $-b/a$) gives 0; reporting the maximising $c = \frac{\sqrt{2}}{4}$ itself gives (A); using $C = -2$ from memory of the $y(\pi/3) = 0$ version gives $\frac{1}{8}$.

Q.69

Application of derivatives – extrema of an integral function • Challenging • 180 s **Answer: A**

Let $f(x) = \displaystyle\int_0^x (t^2 - 5t + 6)\,e^t\,dt$, $x \in \mathbb{R}$. If M and m are respectively the local maximum value and the local minimum value of f, then $M - m$ is equal to

(A) $e^2(3 - e)$ (B) $e^2(e - 2)$ (C) $e^3 - e^2$ (D) $3e^2 - 2e^3$

Solution

$f'(x) = (x - 2)(x - 3)e^x$. This changes sign from $+$ to $-$ at $x = 2$ (local maximum) and from $-$ to $+$ at $x = 3$ (local minimum). So

$$M - m = f(2) - f(3) = -\int_2^3 (t^2 - 5t + 6)e^t\,dt.$$

By repeated parts, $\displaystyle\int P(t)e^t\,dt = e^t\,(P - P' + P'')$. With $P = t^2 - 5t + 6$, $P' = 2t - 5$ and $P'' = 2$:

$$\int (t^2 - 5t + 6)e^t\,dt = e^t\,(t^2 - 7t + 13)\,.$$

Evaluating,

$$\int_2^3 = e^3(9 - 21 + 13) - e^2(4 - 14 + 13) = e^3 - 3e^2.$$

Hence

$$M - m = 3e^2 - e^3 = e^2(3 - e)\ (> 0).$$

Answer: (A)

Common mistake: Antiderivative error $e^t(P + P')$ instead of $e^t(P - P' + P'')$, or computing $f(3) - f(2)$ (a negative number).

Q.70

Definite integration – integral equation, odd/even parts (List match) • Challenging • 200 s **Answer: B**

Let $f(x) = 2\sin x + \displaystyle\int_{-\pi/2}^{\pi/2} (\sin x + t\cos x)\,f(t)\,dt$ for all real x. Match List-I with List-II.

List-I	List-II
(P) $\alpha + \beta$, where $[\alpha, \beta]$ is the range of f	(1) 0
(Q) $f(3\pi/2)$	(2) $2/3$
(R) $f(\pi)$	(3) $4/3$
(S) $f(5\pi/4)$	(4) $\sqrt{2}$
	(5) $-2/3$

Choose the correct option.

(A) (P)–1, (Q)–5, (R)–3, (S)–4 (B) (P)–1, (Q)–2, (R)–3, (S)–4

(C) (P)–2, (Q)–1, (R)–3, (S)–4 (D) (P)–4, (Q)–2, (R)–3, (S)–1

Solution

Write $f(x) = (2+A)\sin x + B\cos x$, where

$$A = \int_{-\pi/2}^{\pi/2} f(t)\,dt, \qquad B = \int_{-\pi/2}^{\pi/2} t\,f(t)\,dt.$$

On $[-\pi/2, \pi/2]$: $\int \sin t = 0$, $\int \cos t = 2$, $\int t\sin t = 2$ and $\int t\cos t = 0$ (odd integrands vanish). So

$$A = 2B, \qquad B = 2(2+A).$$

Then $A = 4(2+A)$, so $A = -\frac{8}{3}$ and $B = -\frac{4}{3}$. Hence

$$f(x) = -\frac{2}{3}\sin x - \frac{4}{3}\cos x.$$

- **(P)** The amplitude is $\sqrt{\frac{4}{9}+\frac{16}{9}} = \frac{2\sqrt{5}}{3}$, so the range is $\left[-\frac{2\sqrt{5}}{3}, \frac{2\sqrt{5}}{3}\right]$ and $\alpha+\beta = 0$: **1**.
- **(Q)** $f(3\pi/2) = -\frac{2}{3}(-1) - 0 = \frac{2}{3}$: **2**.
- **(R)** $f(\pi) = 0 - \frac{4}{3}(-1) = \frac{4}{3}$: **3**.
- **(S)** $f(5\pi/4) = -\frac{2}{3}\left(-\frac{1}{\sqrt{2}}\right) - \frac{4}{3}\left(-\frac{1}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$: **4**.

Answer: (B)

Common mistake: Forgetting that $\int_{-\pi/2}^{\pi/2} t\sin t\,dt = 2$ is non-zero (even integrand), or the sign of $\sin\frac{3\pi}{2}$.

Q.71

Permutations – divisibility by 275 (rules for 25 and 11) • Standard • 240 s **Answer: 180**

The number of eight-digit numbers that can be formed using each of the digits $1, 2, 3, 4, 5, 6, 7, 8$ exactly once, and that are divisible by 275, is

Solution

$275 = 25 \times 11$.

Divisibility by 25. The last two digits must be 25 or 75 (there is no 0, so 00 and 50 are impossible).

Divisibility by 11. Number the positions 1 to 8 from the left. Let O be the sum of the digits in odd positions $1, 3, 5, 7$ and E the sum in even positions $2, 4, 6, 8$. Then $O + E = 36$ and $O - E \equiv 0 \pmod{11}$. Since $|O - E| \le (8+7+6+5) - (1+2+3+4) = 16$ and $O - E$ is even, we need $O - E = 0$, so $O = E = 18$.

Case 1: ends in 25. Position 7 holds 2 and position 8 holds 5. The even positions $2, 4, 6$ need three digits summing to 13 from $\{1, 3, 4, 6, 7, 8\}$, namely $\{1, 4, 8\}$ and $\{3, 4, 6\}$. Odd positions $1, 3, 5$ then need sum 16: the complements $\{3, 6, 7\}$ and $\{1, 7, 8\}$ both work. That is 2 splits, each arranged in $3! \cdot 3! = 36$ ways: 72 numbers.

Case 2: ends in 75. Position 7 holds 7 and position 8 holds 5. The even positions need three digits summing to 13 from $\{1, 2, 3, 4, 6, 8\}$: $\{1, 4, 8\}, \{2, 3, 8\}, \{3, 4, 6\}$. Their complements sum to 11, as required. That is $3 \times 36 = 108$ numbers.

Total $= 72 + 108 = 180$. (Brute force over all $8!$ arrangements confirms 180.)

Answer: 180

Common mistake: Allowing $O - E = \pm 11$ (impossible here because $O + E = 36$ is even), or forgetting the ending 75.

Q.72

3D geometry – plane through a line parallel to another; image; shortest distance • Standard • 210 s **Answer: 37**

Consider the lines $L_1 : \vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(\hat{i}-\hat{j}+2\hat{k})$ and $L_2 : \vec{r} = (2\hat{i}+\hat{k}) + \mu(2\hat{i}+\hat{j}+\hat{k})$. Let d be the shortest distance between L_1 and L_2. Let Π be the plane containing L_1 and parallel to L_2, and let B be the image of the point $A(1,1,1)$ in Π. Then the value of $3d^2 + AB^2$ is

Solution

Direction vectors: $\vec{d}_1 = (1, -1, 2)$ and $\vec{d}_2 = (2, 1, 1)$.

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = (-3, 3, 3) \parallel (1, -1, -1).$$

Shortest distance. $\vec{P}_2 - \vec{P}_1 = (2,0,1) - (1,2,3) = (1, -2, -2)$, so

$$d = \frac{|(1,-2,-2)\cdot(-3,3,3)|}{3\sqrt{3}} = \frac{|-3-6-6|}{3\sqrt{3}} = \frac{5}{\sqrt{3}}, \qquad 3d^2 = 25.$$

Plane Π. Normal $(1,-1,-1)$, passing through $(1,2,3)$:

$$x-y-z+4=0.$$

Image. Distance of $A(1,1,1)$ from Π is $\dfrac{|1-1-1+4|}{\sqrt{3}}=\sqrt{3}$. So $AB=2\sqrt{3}$ and $AB^2=12$. (Explicitly, $B=(-1,3,3)$.)

Therefore $3d^2+AB^2=25+12=37$.

Answer: 37

Common mistake: Using the distance of A from Π instead of AB (twice that distance), which gives 28.

Q.73

Conics – common tangents of a parabola and an ellipse • Challenging • 240 s **Answer: 120**

The parabola $y^2=8x$ and the ellipse $x^2+4y^2=32$ have exactly two common tangents. The four points at which these common tangents touch the two curves are the vertices of a quadrilateral. The area of this quadrilateral (in square units) is

Solution

Any tangent to $y^2=8x$ (here $a=2$) can be written $y=mx+\dfrac{2}{m}$.

The ellipse is $\dfrac{x^2}{32}+\dfrac{y^2}{8}=1$. The line $y=mx+c$ touches it when $c^2=32m^2+8$. So

$$\frac{4}{m^2}=32m^2+8 \;\Rightarrow\; 8m^4+2m^2-1=0 \;\Rightarrow\; (4m^2-1)(2m^2+1)=0 \;\Rightarrow\; m=\pm\frac{1}{2}.$$

The common tangents are $y=\pm\left(\frac{x}{2}+4\right)$.

Points of contact.

- Parabola, $\left(\frac{a}{m^2},\frac{2a}{m}\right)$: $(8,8)$ and $(8,-8)$.
- Ellipse, $\left(-\frac{a^2m}{c},\frac{b^2}{c}\right)$ with $a^2=32,\ b^2=8$: for $m=\frac{1}{2}, c=4$ this is $(-4,2)$; for the other tangent it is $(-4,-2)$.

(Check: $(-4,2)$ satisfies $16+16=32$ and $2=-2+4$.)

The quadrilateral has vertical parallel sides of lengths 16 (at $x=8$) and 4 (at $x=-4$), a distance 12 apart:

$$\text{Area}=\frac{1}{2}(16+4)(12)=120.$$

Answer: 120

Common mistake: Using the parabola tangent $y=mx+\frac{1}{m}$ (that is, $a=1$), or the ellipse condition $c^2=a^2m^2+b^2$ with the semi-axes swapped.

Q.74

Area under curves – functional equation and an improper region • Challenging • 240 s **Answer: 61**

Let $f:(0,\infty)\to\mathbb{R}$ be a differentiable function satisfying $f(xy)=f(x)+f(y)$ for all $x,y>0$, with $f'(1)=2$. Let $g(x)=x^3-7x^2+14x-8$. If $\mathcal{A}$ is the area of the region $\{(x,y):0<x\le 1,\ f(x)\le y\le |g(x)|\}$, then $12\mathcal{A}$ is equal to

Solution

Finding f. Putting $x=y=1$ gives $f(1)=0$. Differentiating $f(xy)=f(x)+f(y)$ with respect to y and then putting $y=1$:

$$xf'(x)=f'(1)=2 \;\Rightarrow\; f(x)=2\ln x+C,$$

and $f(1)=0$ gives $C=0$. So $f(x)=2\ln x$.

Sign of g. $g(x)=(x-1)(x-2)(x-4)$, so $g<0$ on $(0,1)$ and $|g|=-g$ there. Also $2\ln x\le 0\le|g|$ on $(0,1]$, and both curves pass through $(1,0)$.

Area.

$$\mathcal{A}=\int_0^1(-x^3+7x^2-14x+8)\,dx-2\int_0^1\ln x\,dx.$$

The first integral is $-\frac{1}{4}+\frac{7}{3}-7+8=\frac{37}{12}$. For the second (improper but convergent), $\int_0^1\ln x\,dx=[x\ln x-x]_0^1=-1$.

$$\mathcal{A}=\frac{37}{12}+2=\frac{61}{12},\qquad 12\mathcal{A}=61.$$

Answer: 61

Common mistake: Integrating g instead of $|g|$ on $(0,1)$, or dropping the $-\int_0^1 2\ln x\,dx=2$ part as if the log curve were the x-axis.

Q.75 Functions – functional equation with a binomial sum • Challenging • 240 s **Answer: 117**

Let $f:\mathbb{R}\to\mathbb{R}$ be an odd function with $f(1)=1$ and $f\left(\dfrac{x}{x-1}\right)=x\,f(x)$ for every $x<0$. Let

$$S=f(1)f\left(\frac{1}{60}\right)+f\left(\frac{1}{2}\right)f\left(\frac{1}{59}\right)+f\left(\frac{1}{3}\right)f\left(\frac{1}{58}\right)+\cdots+f\left(\frac{1}{30}\right)f\left(\frac{1}{31}\right).$$

If $S=\dfrac{2^K}{M!}$ for positive integers K and M, with M as small as possible, then $K+M$ is equal to

Solution

Put $x=-\frac{1}{k}$ ($k\in\mathbb{N}$, so $x<0$). Then

$$\frac{x}{x-1}=\frac{-1/k}{-1/k-1}=\frac{1}{k+1}.$$

So

$$f\left(\frac{1}{k+1}\right)=-\frac{1}{k}f\left(-\frac{1}{k}\right)=\frac{1}{k}f\left(\frac{1}{k}\right)\quad(f\text{ odd}).$$

With $f(1)=1$, induction gives $f\left(\dfrac{1}{n}\right)=\dfrac{1}{(n-1)!}$.

The i-th term of S is $f\left(\frac{1}{i}\right)f\left(\frac{1}{61-i}\right)$, for $i=1,\dots,30$:

$$S=\sum_{i=1}^{30}\frac{1}{(i-1)!\,(60-i)!}=\frac{1}{59!}\sum_{j=0}^{29}\binom{59}{j}=\frac{1}{59!}\cdot\frac{2^{59}}{2}=\frac{2^{58}}{59!},$$

using the symmetry of $\binom{59}{j}$ (the 60 coefficients split into two equal halves).

Uniqueness. If $2^K/M!=2^{58}/59!$ with $M\neq 59$, then $M!/59!$ (or its reciprocal) would be a power of 2. That is impossible: for $M<59$ the ratio $59!/M!$ contains the odd prime 59, and for $M>59$ the ratio $M!/59!$ contains the factor 60, which is divisible by 3. So $K=58$, $M=59$ and $K+M=117$. (An exact computer check confirms $S=2^{58}/59!$.)

Answer: 117

Common mistake: Getting $f(1/n)=1/n!$ (an off-by-one in the induction), or summing j from 0 to 30 instead of 29.