॥आचार्य सारथी॥
ACHARYA SARTHI
प्रत्येक विद्यार्थ्याचा सारथी
Every Indian student's guide
acharyasarthi.com • Free practice papers for Maharashtra families

JEE Main Free Mock 1

JEE (Main) 2027 Pattern · Full Syllabus · Free mock test

Date	Practise any day (3 hours)	**Duration**	3 Hours (180 minutes)
Maximum Marks	300 (100 per subject)	**Questions**	75 (25 per subject)

Student Name		**Roll No. / ID**	
School / Class		**Parent's sign**	

Marking scheme (each subject)

Section	Question type	Qs	Marks	Max
A	Single correct option (MCQ)	20	+4 correct, −1 wrong, 0 unattempted	80
B	Numerical value (integer)	5	+4 correct, −1 wrong, 0 unattempted	20
			Total per subject	**100**

Instructions

1. The paper has three parts: Part A Physics (Q.1–25), Part B Chemistry (Q.26–50) and Part C Mathematics (Q.51–75). Each part has Section A (20 MCQs) and Section B (5 numerical-value questions).
2. All questions are compulsory. Each MCQ has exactly one correct option.
3. The answer to every numerical-value question is an integer; it may be negative. Do not enter units.
4. Unless stated otherwise take $g = 10\ \mathrm{m\,s^{-2}}$. Figures marked "Schematic" are not to scale.
5. Use of calculator, mobile phone or any electronic device is not allowed. Use the space provided for rough work.

PART A : PHYSICS

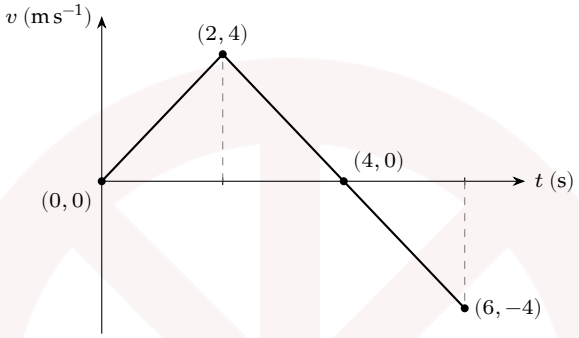
25 Questions • 100 Marks

SECTION A • Multiple choice questions

This section contains TWENTY (20) questions. Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
Full marks: +4 if only the correct option is chosen; Zero marks: 0 if no option is chosen; Negative marks: −1 in all other cases.

Q.1 A resistive force is written $F = av + bv^2$, where v is speed. What are the dimensions of a/b?
(A) $\mathrm{M\,L^{-1}}$ (B) $\mathrm{L^2\,T^{-2}}$ (C) $\mathrm{L\,T^{-1}}$ (D) $\mathrm{T^{-1}}$

Q.2 The plotted velocity of a particle joins $(0,0)$, $(2,4)$, $(4,0)$, $(6,-4)$, with t in s and v in $\mathrm{m\,s^{-1}}$. Its distance travelled divided by the magnitude of displacement in 6 s is

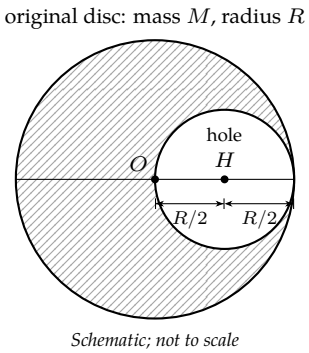


(A) 1 (B) 3 (C) 4 (D) 2

Q.3 Two blocks of masses 2 kg and 3 kg touch on a smooth horizontal floor. A horizontal 10 N force pushes the 2 kg block toward the 3 kg block. The contact force is
(A) 4 N (B) 10 N (C) 15 N (D) 6 N

Q.4 A 1 kg cart starts at rest at $x = 0$. Its net horizontal force varies linearly from 4 N at $x = 0$ to −2 N at $x = 3$ m. Its speed at $x = 3$ m is
(A) $\sqrt{6}\,\mathrm{m\,s^{-1}}$ (B) 0 (C) $3\,\mathrm{m\,s^{-1}}$ (D) $\sqrt{12}\,\mathrm{m\,s^{-1}}$

Q.5 A uniform disc of original mass M and radius R has an internally tangent circular hole of radius $R/2$ removed, with its centre H at distance $R/2$ to the right of the original centre O. The remaining lamina rotates about an axis through its own centre of mass perpendicular to its plane. Its moment of inertia is

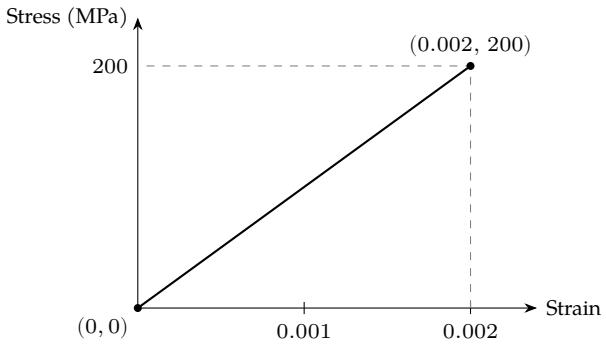


Schematic; not to scale

(A) $\dfrac{37MR^2}{96}$ (B) $\dfrac{3MR^2}{8}$
(C) $\dfrac{13MR^2}{32}$ (D) $\dfrac{5MR^2}{12}$

Q.6 At what altitude above a spherical planet of radius R is gravitational acceleration one quarter of its surface value?
(A) $2R$ (B) $R/2$ (C) R (D) $3R$

Q.7 A wire has the linear stress–strain graph through $(0, 0)$ and $(0.002,\ 200\ \text{MPa})$, and volume $2 \times 10^{-6}\ \text{m}^3$. It is first loaded to strain 0.001 and then stretched quasistatically to 0.002. The additional work in this second stage is

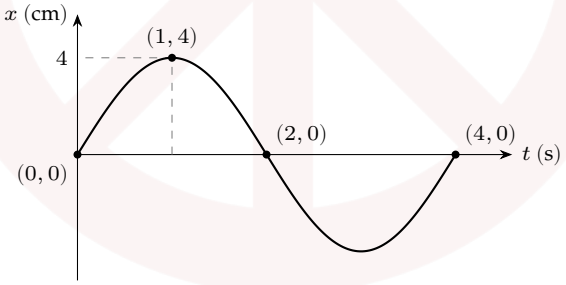


(A) 0.2 J (B) 0.1 J (C) 0.3 J (D) 0.4 J

Q.8 During compression a gas releases 30 J of heat while 80 J of work is done on it. Its internal energy changes by
(A) -50 J (B) $+50$ J (C) $+110$ J (D) -110 J

Q.9 Equal numbers of molecules of a monatomic and a rigid diatomic ideal gas are at the same temperature. Neglect vibration. Their total internal energies are in the ratio
(A) $5 : 3$ (B) $3 : 5$ (C) $3 : 7$ (D) $1 : 1$

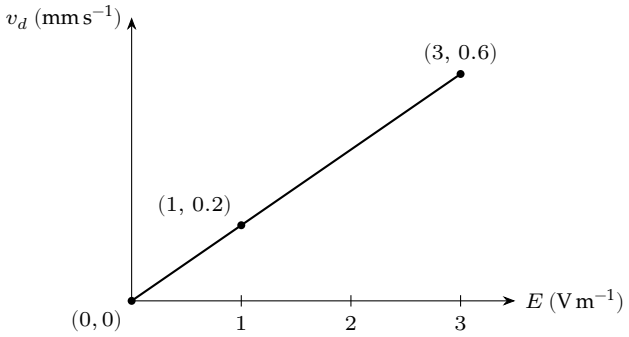
Q.10 An SHM trace starts at $x = 0$ moving toward positive x. Its FIRST positive maximum, $+4$ cm, occurs at $t = 1$ s and its NEXT zero at $t = 2$ s. Between $t = 0$ and $t = 7/3$ s, the total distance travelled divided by the magnitude of displacement is



(A) 7 (B) 4 (C) 3 (D) 5

Q.11 Positive charges Q and $4Q$ are fixed 30 cm apart. A small positive test charge experiences zero net force between them at a distance from Q of
(A) 15 cm (B) 6 cm (C) 20 cm (D) 10 cm

Q.12 A conductor has the measured drift-speed graph shown. A 2 m specimen of it carries current I when connected to a 6 V supply. It is replaced by a 4 m specimen of the same material and temperature with half the cross-sectional area, connected to the same supply. The new current is



(A) $2I$ (B) $I/4$ (C) I (D) $I/2$

Q.13 A long solenoid has N turns over length L and carries current I. Keeping N and I fixed, it is stretched to length $2L$ while its radius remains much smaller than L. A second coaxial long solenoid provides an opposing field equal to one quarter of the original first-solenoid field. The net interior field magnitude is

(A) three quarters of the original first-solenoid field

(B) one half of the original first-solenoid field

(C) one quarter of the original first-solenoid field

(D) zero

Q.14 A horizontal conducting ring of area 0.02 m^2 and resistance $2\ \Omega$ is viewed from above. Its area is increasing at 0.01 m^2 s^{-1} while an upward field of 0.4 T is decreasing at 0.1 T s^{-1}. At that instant the induced current is

(A) 1 mA anticlockwise (B) 3 mA anticlockwise

(C) 1 mA clockwise (D) 3 mA clockwise

Q.15 A parallel-plate capacitor of circular plate radius a is charged at constant current I. Neglect fringing. For coaxial circular loops of radii $a/2$ and $2a$ lying in a plane between the plates (the loop of radius $2a$ extends beyond the plate edge), the ratio of magnetic fields $B(a/2)/B(2a)$ is

(A) 4 (B) 1/4 (C) 2 (D) 1

Q.16 An object is placed 30 cm in front of a concave mirror of focal length 15 cm. Its image is

(A) real, inverted and equal in size (B) virtual, erect and enlarged

(C) at infinity (D) real, inverted and half-sized

Q.17 A photocathode gives stopping potentials 1 V and 3 V for photon energies 3 eV and 5 eV. It is next illuminated by 4 eV photons and a retarding potential of 0.5 V is applied. The largest electron kinetic energy at the collector is

(A) 1.5 eV (B) 2.5 eV (C) 0.5 eV (D) 2 eV

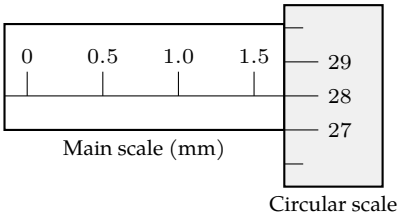
Q.18 Node X is connected to $+10$ V through 2 kΩ and to ground through 3 kΩ. Two ideal diodes connect X to reference supplies: D_1 has anode X and cathode $+5$ V; D_2 has anode $+2$ V and cathode X. The steady current through D_1 is

(A) $\frac{5}{3}$ mA (B) $\frac{5}{6}$ mA

(C) $\frac{5}{2}$ mA (D) 0

Q.19 The screw-gauge scale shown has pitch 0.5 mm and 50 circular divisions; its closed-jaw reading is $+0.03$ mm. This wire gives main reading 1.5 mm and circular reading 28. A second wire measured with the same gauge gives main reading 1.0 mm and circular reading 43. The ratio of corrected diameters, first to second, is

Pitch 0.5 mm; 50 divisions; zero error $+0.03$ mm



Schematic; not to scale

(A) $\dfrac{178}{143}$ (B) $\dfrac{4}{5}$

(C) $\dfrac{7}{5}$ (D) $\dfrac{5}{4}$

Q.20 A 2 kg block on a horizontal floor has $\mu_s = 0.5$ and $\mu_k = 0.3$. A force is applied at 37° above horizontal ($\cos 37^\circ = 0.8$, $\sin 37^\circ = 0.6$); take $g = 10\ \mathrm{m\,s^{-2}}$. The force is raised slowly to the threshold for sliding and is then held at that magnitude. The acceleration just after sliding begins is

(A) $\dfrac{16}{11}\ \mathrm{m\,s^{-2}}$ (B) $\dfrac{20}{11}\ \mathrm{m\,s^{-2}}$

(C) $\dfrac{8}{11}\ \mathrm{m\,s^{-2}}$ (D) 0

SECTION B • Numerical value questions

This section contains FIVE (05) questions. The answer to each question is an INTEGER (it may be negative). All five are compulsory.
Full marks: +4 if only the correct value is entered; Zero marks: 0 if unattempted; Negative marks: −1 in all other cases.

Q.21 Hydrogen atoms are excited to $n = 5$ and all downward cascades occur in a large collection. An ideal filter removes every photon from a transition ending at $n = 1$ or $n = 2$. How many distinct spectral lines pass through the filter?

Answer: _____

Q.22 A measured volume is calculated as $(12.11\ \mathrm{cm} + 0.3\ \mathrm{cm} + 1.024\ \mathrm{cm}) \times 2.0\ \mathrm{cm} \times 1.00\ \mathrm{cm}$. Use normal significant-figure rules, retaining guard digits until the end. Enter the number of significant figures in the final volume.

Answer: _____

Q.23 A boat moves at $5\ \mathrm{m\,s^{-1}}$ relative to still water. A river flows east at $3\ \mathrm{m\,s^{-1}}$. The boat is aimed so that it travels due north across a river 80 m wide; find the crossing time in seconds.

Answer: _____

Q.24 A 2 kg particle has potential energy $U(x) = 2x^2 - 8x + 10$ J, with x in metres. Released from rest at $x = 0$, what is its maximum kinetic energy in joules?

Answer: _____

Q.25 A turntable of inertia $2\ \mathrm{kg\,m^2}$ rotates at $6\ \mathrm{rad\,s^{-1}}$ about a frictionless vertical axle. A 1 kg lump moving horizontally at $1.5\ \mathrm{m\,s^{-1}}$ tangentially in the same rotational sense strikes and sticks at radius 1 m. It is then pulled slowly to radius 0.5 m without external torque about the axle. Find the final angular speed in $\mathrm{rad\,s^{-1}}$.

Answer: _____

PART B : CHEMISTRY

25 Questions • 100 Marks

SECTION A • Multiple choice questions

This section contains TWENTY (20) questions. Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
Full marks: +4 if only the correct option is chosen; Zero marks: 0 if no option is chosen; Negative marks: −1 in all other cases.

Q.26 Consider the following statements.
Statement I: For $2\,H_2 + O_2 \longrightarrow 2\,H_2O$, 3 mol H_2 mixed with 2 mol O_2 makes H_2 the limiting reagent.
Statement II: At 80% yield, this mixture produces 2.4 mol water.
Choose the correct option.
(A) I is correct; II is incorrect
(B) Both I and II are incorrect
(C) I is incorrect; II is correct
(D) Both I and II are correct

Q.27 An electron in a hydrogen atom falls from $n = 4$ to $n = 2$. The same photon can ionise an excited hydrogen atom initially in which lowest principal quantum level?
(A) 2 (B) 4 (C) 3 (D) 1

Q.28 Consider the following statements.
Statement I: In an octet-obeying Lewis structure of CO, carbon has formal charge -1.
Statement II: In that same structure, oxygen has formal charge $+1$.
Choose the correct option.
(A) Both I and II are incorrect
(B) I is incorrect; II is correct
(C) I is correct; II is incorrect
(D) Both I and II are correct

Q.29 **Assertion (A):** An isolated ideal gas expanding freely into vacuum has $\Delta U = 0$.
Reason (R): During free expansion into vacuum the gas performs positive pressure–volume work on its surroundings.
Choose the correct option.
(A) Both A and R are true and R correctly explains A
(B) Both A and R are true but R does not correctly explain A
(C) A is true but R is false
(D) A is false but R is true

Q.30 A 40% by mass aqueous urea stock has density $1.20\ \mathrm{g\,mL^{-1}}$. A student takes 50.0 mL, adds 40.0 g water, and evaporates 20.0 g water without losing urea. The final density is $1.10\ \mathrm{g\,mL^{-1}}$. With molar mass $60\ \mathrm{g\,mol^{-1}}$, choose final molarity and molality.
(A) 5.50 M and $7.14\ \mathrm{mol\,kg^{-1}}$
(B) 4.40 M and $5.00\ \mathrm{mol\,kg^{-1}}$
(C) 7.14 M and $5.50\ \mathrm{mol\,kg^{-1}}$
(D) 5.50 M and $5.00\ \mathrm{mol\,kg^{-1}}$

Q.31 Consider the following statements.
Statement I: For $H_2(g) + I_2(g) \rightleftharpoons 2\,HI(g)$, K_p and K_c have equal numerical values under the usual ideal-gas convention.
Statement II: For $N_2O_4(g) \rightleftharpoons 2\,NO_2(g)$, $K_p/K_c = RT$ under that convention.
Choose the correct option.
(A) Both I and II are incorrect
(B) I is incorrect; II is correct
(C) Both I and II are correct
(D) I is correct; II is incorrect

Q.32 A solution initially contains 0.050 mol Fe^{2+}. It is treated with 0.010 mol $Cr_2O_7^{2-}$ in acid. The excess dichromate is then consumed by added Fe^{2+}. How many millimoles of additional Fe^{2+} are required?
(A) 60 (B) 10 (C) 20 (D) 5

Q.33 Consider the following statements.
Statement I: If doubling [A] at fixed [B] multiplies the initial rate by four, the order in A is two.
Statement II: If doubling [B] at fixed [A] leaves the rate unchanged, B must be absent from the balanced chemical equation.
Choose the correct option.
(A) I is correct; II is incorrect
(B) Both I and II are correct
(C) Both I and II are incorrect
(D) I is incorrect; II is correct

Q.34 Consider the following statements.
Statement I: Among O^{2-}, F^-, Na^+ and Mg^{2+}, Mg^{2+} has the smallest ionic radius.
Statement II: The same four ions all contain ten electrons.
Choose the correct option.
(A) Both I and II are incorrect
(B) Both I and II are correct
(C) I is incorrect; II is correct
(D) I is correct; II is incorrect

Q.35 Consider the following statements.
Statement I: The stability of the $+1$ oxidation state generally increases from aluminium to thallium.
Statement II: Boron commonly forms a simple B^{3+} ion in aqueous solution because of its small size.
Choose the correct option.
(A) I is incorrect; II is correct
(B) I is correct; II is incorrect
(C) Both I and II are incorrect
(D) Both I and II are correct

Q.36 Which ion has a $3d^5$ configuration? Atomic numbers: Cr 24, Mn 25, Fe 26, Co 27.
(A) Fe^{2+} (B) Cr^{3+} (C) Co^{2+} (D) Mn^{2+}

Q.37 Match List I with List II.

List I	List II
P. $K_4[Fe(CN)_6]$	1. Diamminedichloridoplatinum(II)
Q. $[Co(NH_3)_6]Cl_3$	2. Potassium hexacyanidoferrate(II)
R. $[Pt(NH_3)_2Cl_2]$	3. Hexaamminecobalt(III) chloride

Choose the correct option.
(A) P-1, Q-2, R-3 (B) P-2, Q-1, R-3 (C) P-2, Q-3, R-1 (D) P-1, Q-3, R-2

Q.38 **Assertion (A):** A small amount of hot solvent is usually preferable to a large excess for recrystallising a solid with strongly temperature-dependent solubility.
Reason (R): Using excess solvent can leave more product dissolved in the cold mother liquor.
Choose the correct option.
(A) Both A and R are true and R correctly explains A
(B) Both A and R are true but R does not correctly explain A
(C) A is true but R is false
(D) A is false but R is true

Q.39 Match List I with List II.

List I	List II
P. CH_3CH_2CHO	1. Carboxylic acid; propanoic acid
Q. CH_3COCH_3	2. Aldehyde; propanal
R. CH_3CH_2COOH	3. Ketone; propan-2-one

Choose the correct option.
(A) P-2, Q-1, R-3 (B) P-2, Q-3, R-1 (C) P-1, Q-3, R-2 (D) P-1, Q-2, R-3

Q.40 For ethane, let the torsional potential be $V(\theta) = \frac{V_0}{2}(1 + \cos 3\theta)$, with θ measured by rotation of one CH_3 group relative to the other about the C–C bond, and $\theta = 0^\circ$ fully eclipsed. Over $0^\circ < \theta < 240^\circ$, how many stationary conformations occur, and how many are minima?
(A) 3 and 2 (B) 4 and 2 (C) 3 and 1 (D) 4 and 3

Q.41 For methyl halides reacting by S_N2 with the same nucleophile in the same solvent, which generally reacts fastest?
(A) CH_3Br (B) CH_3Cl (C) CH_3F (D) CH_3I

Q.42 Which alcohol gives turbidity most rapidly with Lucas reagent at room temperature?
(A) Propan-1-ol (B) Propan-2-ol
(C) Ethanol (D) 2-Methylpropan-2-ol

Q.43 Match List I with List II.

List I	List II
P. CH_3CONH_2 with Br_2/NaOH	1. Ethylamine
Q. CH_3CN with $LiAlH_4$, then water	2. Aniline
R. $C_6H_5NO_2$ with Sn/HCl, then base	3. Methylamine

Choose the correct option.
(A) P-3, Q-1, R-2 (B) P-2, Q-1, R-3 (C) P-1, Q-2, R-3 (D) P-1, Q-3, R-2

Q.44 Which disaccharide gives only D-glucose on complete hydrolysis?
(A) Maltose (B) Lactose (C) Sucrose (D) All three

Q.45 Consider the following statements.
Statement I: Crystallising Mohr salt from 0.10 mol $FeSO_4$ and 0.08 mol $(NH_4)_2SO_4$ has a maximum theoretical yield of 0.08 mol $(NH_4)_2Fe(SO_4)_2 \cdot 6H_2O$, if all required water is available.
Statement II: Crystallising potash alum from 0.10 mol K_2SO_4 and 0.08 mol $Al_2(SO_4)_3$ has a maximum theoretical yield of 0.08 mol $KAl(SO_4)_2 \cdot 12H_2O$.
Choose the correct option. Assume sufficient water and no side reactions in both cases.
(A) I is incorrect; II is correct (B) Both I and II are correct
(C) I is correct; II is incorrect (D) Both I and II are incorrect

SECTION B • Numerical value questions

This section contains FIVE (05) questions. The answer to each question is an INTEGER (it may be negative). All five are compulsory.
Full marks: +4 if only the correct value is entered; Zero marks: 0 if unattempted; Negative marks: −1 in all other cases.

Q.46 An organic compound has empirical formula CH_2O and molar mass $180\ g\,mol^{-1}$. How many oxygen atoms occur in one molecule? Use C = 12, H = 1, O = 16.
Answer: _____

Q.47 How many electrons can occupy all orbitals having $n = 4$ and $l = 2$?
Answer: _____

Q.48 For $MX_2(s)$, $\Delta H_f^\circ = -600\ kJ\,mol^{-1}$ and lattice dissociation enthalpy is $+2100\ kJ\,mol^{-1}$. Atomising M(s) costs $150\ kJ\,mol^{-1}$, breaking $X_2(g)$ costs $200\ kJ\,mol^{-1}$, and each X atom gains an electron with $\Delta H = -300\ kJ\,mol^{-1}$. If the first ionisation enthalpy of M is $500\ kJ\,mol^{-1}$, find its second ionisation enthalpy in $kJ\,mol^{-1}$.
Answer: _____

Q.49 Given ΔH_f° of $CO_2(g)$, $H_2O(l)$ and $CH_4(g)$ as -394, -286 and $-75\ kJ\,mol^{-1}$, find the magnitude of methane combustion enthalpy in $kJ\,mol^{-1}$.
Answer: _____

Q.50 A binary mixture contains an overall mole fraction 0.36 of A and forms coexisting ideal liquid and vapour phases. At the equilibrium temperature, $p_{\mathrm{A}}^{\circ} = 350$ kPa and $p_{\mathrm{B}}^{\circ} = 100$ kPa, and total pressure is 175 kPa. What percentage of the total moles is in the vapour phase?

Answer: _____

PART C : MATHEMATICS

25 Questions • 100 Marks

SECTION A • Multiple choice questions

This section contains TWENTY (20) questions. Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
Full marks: +4 if only the correct option is chosen; Zero marks: 0 if no option is chosen; Negative marks: −1 in all other cases.

Q.51 In a class of 60, 35 study French and 28 study German; 10 study neither. How many study exactly one of these languages?
(A) 38 (B) 39 (C) 37 (D) 36

Q.52 If $z = \dfrac{1+i\sqrt{3}}{\sqrt{3}-i}$, find the principal argument of z.
(A) $\dfrac{\pi}{2}$ (B) $\dfrac{2\pi}{3}$
(C) $-\dfrac{\pi}{2}$ (D) $\dfrac{\pi}{6}$

Q.53 $A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$. Find the sum of the off-diagonal entries of A^2.
(A) 7 (B) 6 (C) 4 (D) 5

Q.54 Six distinct books, including a physics and a mathematics book, are arranged on a shelf. How many arrangements have exactly two books between these two specified books?
(A) 144 (B) 240 (C) 288 (D) 72

Q.55 In $\left(x^2 + \dfrac{1}{x}\right)^n$, n is a positive integer. The ratio of the coefficient of x^6 to that of x^3 is $\dfrac{3}{2}$, and both terms occur. Find n.
(A) 12 (B) 18 (C) 6 (D) 24

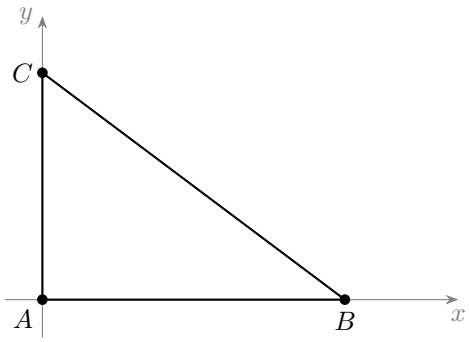
Q.56 A positive geometric progression has second term 6 and fourth term 24. Find its fifth term.
(A) 12 (B) 48 (C) 96 (D) 36

Q.57 Find the number of integers x in the domain of $\dfrac{\sqrt{(x-1)(5-x)}}{\ln(x-2)}$.
(A) 3 (B) 4 (C) 1 (D) 2

Q.58 Evaluate $\displaystyle\int_0^1 x\,e^x\,dx$.
(A) 2 (B) 0 (C) 1 (D) 3

Q.59 For $\left(\dfrac{d^2y}{dx^2}\right)^3 + \sqrt{1+\left(\dfrac{dy}{dx}\right)^2} = 0$, eliminating the radical produces a polynomial differential equation. What is its degree?
(A) 6 (B) 8 (C) 5 (D) 7

Q.60 $A=(0,0)$, $B=(8,0)$, $C=(0,6)$. P on BC satisfies $BP:PC=1:3$. The perpendicular bisector of AP meets the x-axis at T. Find the ratio of the areas of triangles PTC and ABC.



(A) $\frac{231}{256}$ (B) $\frac{231}{512}$

(C) $\frac{77}{256}$ (D) $\frac{17}{128}$

Q.61 $A=(1,-2,3)$ and $B=(5,4,-1)$. Find the square of the distance from the origin to their midpoint.

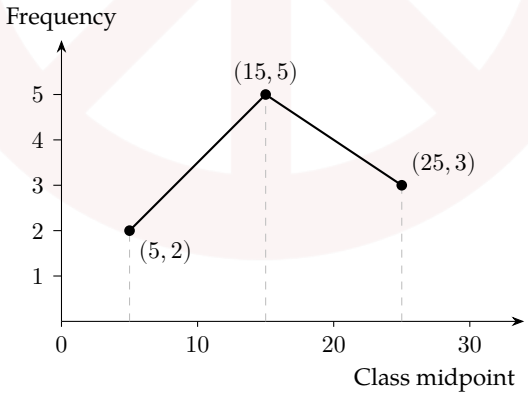
(A) 12 (B) 10 (C) 11 (D) 13

Q.62 Nonparallel vectors $\vec{a},\vec{b}$ determine $\vec{u}=2\vec{a}+\vec{b}$ and $\vec{v}=\vec{a}-\vec{b}$. A point P has position vector $\lambda\vec{u}+(1-\lambda)\vec{v}$. If its $\vec{a}$- and $\vec{b}$-coefficients are equal, find λ.

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$

(C) 2 (D) -2

Q.63 Classes $[0,10)$, $[10,20)$, $[20,30)$ have frequencies $2,5,3$. Using class midpoints, find the mean.



(A) 18 (B) 15 (C) 17 (D) 16

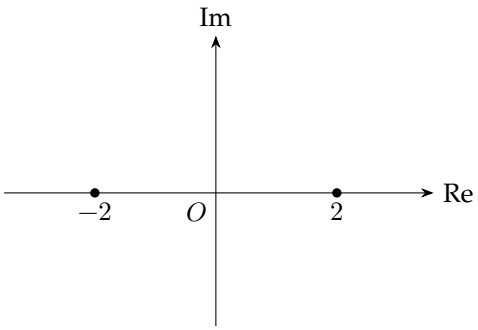
Q.64 For an acute θ, $\sin\theta+\cos\theta=\sqrt{\frac{3}{2}}$. Find $(\tan\theta-\cot\theta)^2$.

(A) 4 (B) 14 (C) 12 (D) 16

Q.65 Sets A and B satisfy $|A|=|B|=5$ and $|A\cap B|=2$. How many ordered pairs (X,Y) satisfy $X\subseteq A$, $Y\subseteq B$, $X\cap Y=\emptyset$ and $|X\cup Y|=3$?

(A) 110 (B) 104 (C) 160 (D) 80

Q.66 If $z = x + iy$ satisfies $|z - 2| = |z + 2|$, which is its locus?



(A) $x = 0$ (B) $y = 0$ (C) $x^2 + y^2 = 4$ (D) $x = 2$

Q.67 Let $A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$. How many real 2×2 matrices X satisfy $AX + XA = \begin{pmatrix} 4 & 0 \\ 0 & -4 \end{pmatrix}$, $\text{tr}(X) = 0$ and $\det(X) = -1$?

(A) 2 (B) Infinitely many
(C) 1 (D) 0

Q.68 From 5 men and 4 women a committee of 4 is chosen. Two specified women refuse to serve together. How many committees contain exactly two women?

(A) 40 (B) 50 (C) 60 (D) 100

Q.69 For positive a, b, the two middle terms of $(a + b)^7$ are equal and their sum is 8960. Find $a + b$.

(A) 8 (B) 4 (C) 16 (D) 2

Q.70 Between positive numbers $a < b$, two geometric means p, q and one arithmetic mean A are inserted. If $p + q = 18$ and $A = 21$, find $b - a$.

(A) 39 (B) 36 (C) 27 (D) 42

SECTION B • Numerical value questions

This section contains FIVE (05) questions. The answer to each question is an INTEGER (it may be negative). All five are compulsory.
Full marks: +4 if only the correct value is entered; Zero marks: 0 if unattempted; Negative marks: −1 in all other cases.

Q.71 Find $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + 4x} - 1 - 2x}{x^2}$. (The answer may be negative.)

Answer: _____

Q.72 Let $I = \displaystyle\int_0^1 \frac{x}{(x + 1)(x + 2)}\, dx$. If $I = \ln \dfrac{a}{b}$ with positive coprime integers a, b, find $a + b$.

Answer: _____

Q.73 A positive solution satisfies $y' = \dfrac{x}{y}$ and $y(0) = 2$. Let $a > 0$ satisfy $y(a) = 2a$. Find $3a^2$.

Answer: _____

Q.74 $P = (x, y)$ satisfies $PA^2 + PB^2 = 50$ for $A = (-3, 0)$, $B = (3, 0)$. A line through the origin with slope 1 meets this locus at U, V. Find $|UV|^2$.

Answer: _____

Q.75 A direction vector is $(1, t, t^2)$. Its angle with the positive y-axis has cosine $\dfrac{1}{\sqrt{3}}$. Let $\hat{u}$ be its unit vector with positive first component, and $\hat{v}$ the unit vector along $(1, -2, 2)$. Find $27\,|\hat{u} \times \hat{v}|^2$.

Answer: _____

— END OF PAPER —