॥आचार्य सारथी॥
ACHARYA SARTHI
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# JEE Main Free Mock 1

Answer Key & Detailed Solutions · Free mock test

## Answer key

| Physics | | | | |
|---|---|---|---|---|
| **1** : C | **2** : B | **3** : D | **4** : A | **5** : A |
| **6** : C | **7** : C | **8** : B | **9** : B | **10** : D |
| **11** : D | **12** : B | **13** : C | **14** : C | **15** : D |
| **16** : A | **17** : A | **18** : B | **19** : D | **20** : A |
| **21** : 3 | **22** : 2 | **23** : 20 | **24** : 8 | **25** : 6 |

| Chemistry | | | | |
|---|---|---|---|---|
| **26** : D | **27** : C | **28** : D | **29** : C | **30** : A |
| **31** : C | **32** : B | **33** : A | **34** : B | **35** : B |
| **36** : D | **37** : C | **38** : A | **39** : B | **40** : A |
| **41** : D | **42** : D | **43** : A | **44** : A | **45** : C |
| **46** : 6 | **47** : 10 | **48** : 1250 | **49** : 891 | **50** : 20 |

| Mathematics | | | | |
|---|---|---|---|---|
| **51** : C | **52** : A | **53** : D | **54** : A | **55** : D |
| **56** : B | **57** : D | **58** : C | **59** : A | **60** : B |
| **61** : C | **62** : C | **63** : D | **64** : C | **65** : B |
| **66** : A | **67** : D | **68** : B | **69** : B | **70** : A |
| **71** : -2 | **72** : 17 | **73** : 4 | **74** : 64 | **75** : 26 |

Q.1–20, 26–45, 51–70: MCQ (+4/−1) • Q.21–25, 46–50, 71–75: integer answers (+4/−1).

## Chapter, difficulty and time map

| Q | Code | Chapter / concept | Level | Time (s) |
|---|---|---|---|---|
| 1 | P01.01 | Dimensional consistency | Foundation | 65 |
| 2 | P02.01 | Displacement-time and velocity-time graphs | Foundation | 115 |
| 3 | P03.01 | Free-body diagrams and connected masses | Foundation | 65 |
| 4 | P04.01 | Work from force-position graph | Standard | 115 |
| 5 | P05.01 | Moment of inertia and axes theorems | Challenging | 185 |
| 6 | P06.01 | Gravity versus altitude and depth | Foundation | 65 |
| 7 | P07.01 | Stress-strain and elastic energy | Standard | 115 |

| | | | | |
|---|---|---|---|---|
| 8 | P08.01 | First-law sign convention | Foundation | 65 |
| 9 | P09.02 | Degrees of freedom | Foundation | 65 |
| 10 | P10.01 | SHM phase and graphs | Standard | 185 |
| 11 | P11.01 | Coulomb force and equilibrium | Foundation | 65 |
| 12 | P12.01 | Drift speed and mobility | Standard | 115 |
| 13 | P13.01 | Field of wire loop and solenoid | Standard | 115 |
| 14 | P14.01 | Flux change and Lenz law | Standard | 115 |
| 15 | P15.01 | Displacement current | Standard | 115 |
| 16 | P16.01 | Spherical mirror image formation | Foundation | 65 |
| 17 | P17.01 | Photoelectric graphs and threshold | Standard | 115 |
| 18 | P19.01 | Diode IV and bias | Standard | 185 |
| 19 | P20.01 | Vernier and screw gauge readings | Standard | 115 |
| 20 | P03.02 | Static versus kinetic friction | Challenging | 185 |
| 21 | P18.01 | Bohr transitions and spectral lines | Standard | 115 |
| 22 | P01.02 | Significant figures | Standard | 115 |
| 23 | P02.02 | Relative motion in a plane | Foundation | 115 |
| 24 | P04.02 | Conservative force and potential energy | Foundation | 115 |
| 25 | P05.02 | Angular momentum conservation | Standard | 185 |
| 26 | C01.01 | Mole concept – limiting reagent and yield | Foundation | 55 |
| 27 | C02.01 | Atomic structure – Bohr energy and spectrum | Standard | 95 |
| 28 | C03.01 | Chemical bonding – Lewis structure and formal charge | Foundation | 55 |
| 29 | C04.01 | Thermodynamics – state functions and process work | Foundation | 95 |
| 30 | C05.01 | Solutions – concentration conversion | Challenging | 145 |
| 31 | C06.01 | Chemical equilibrium – $K$_p–$K$_c conversion | Foundation | 55 |
| 32 | C07.01 | Redox – oxidation number and back-titration | Standard | 145 |
| 33 | C08.01 | Chemical kinetics – rate law from data | Standard | 55 |
| 34 | C09.01 | Periodicity – atomic and ionic radii | Foundation | 95 |
| 35 | C10.01 | p-Block – group 13 trends and first-element anomaly | Foundation | 95 |
| 36 | C11.01 | d-Block – transition configurations and oxidation states | Foundation | 55 |
| 37 | C12.01 | Coordination compounds – nomenclature and formula | Foundation | 95 |
| 38 | C13.01 | Purification – crystallisation | Foundation | 55 |
| 39 | C14.01 | Organic chemistry – IUPAC nomenclature and functional groups | Foundation | 95 |
| 40 | C15.01 | Hydrocarbons – ethane conformations | Challenging | 145 |
| 41 | C16.01 | Haloalkanes – C–X bond and S_N2 reactivity | Foundation | 55 |
| 42 | C17.01 | Alcohols – classification and Lucas test | Foundation | 95 |
| 43 | C18.01 | Amines – preparation | Standard | 95 |
| 44 | C19.02 | Biomolecules – sucrose, lactose and maltose | Foundation | 95 |
| 45 | C20.02 | Practical chemistry – Mohr salt and potash alum preparation | Standard | 95 |
| 46 | C01.02 | Mole concept – empirical and molecular formula | Foundation | 95 |
| 47 | C02.02 | Atomic structure – quantum numbers and orbital capacity | Foundation | 95 |
| 48 | C03.02 | Chemical bonding – Born–Haber lattice enthalpy cycle | Standard | 145 |
| 49 | C04.02 | Thermodynamics – Hess's law and formation enthalpy | Foundation | 95 |
| 50 | C05.02 | Solutions – Raoult's law and vapour–liquid split | Challenging | 145 |
| 51 | M01.01 | Sets – set operations and cardinality | Foundation | 80 |
| 52 | M02.01 | Complex numbers – modulus and argument | Foundation | 80 |
| 53 | M03.01 | Matrices – types and operations | Foundation | 80 |
| 54 | M04.01 | Permutations – constrained arrangements | Standard | 150 |
| 55 | M05.01 | Binomial theorem – general term | Standard | 220 |
| 56 | M06.02 | Sequences – geometric progression | Foundation | 80 |
| 57 | M07.01 | Functions – domain, range and graphs | Standard | 150 |
| 58 | M08.02 | Integration – integration by parts | Foundation | 80 |
| 59 | M09.01 | Differential equations – order and degree | Standard | 150 |
| 60 | M10.01 | Coordinate geometry – distance and section formula | Challenging | 220 |
| 61 | M11.01 | 3D geometry – distance and section in space | Foundation | 80 |
| 62 | M12.01 | Vectors – components and addition | Standard | 150 |
| 63 | M13.01 | Statistics – mean of grouped data | Foundation | 150 |
| 64 | M14.01 | Trigonometry – identity transformations | Standard | 150 |
| 65 | M01.02 | Sets – power sets | Challenging | 220 |
| 66 | M02.02 | Complex numbers – Argand-plane locus | Foundation | 80 |
| 67 | M03.02 | Matrices – matrix equations | Challenging | 150 |
| 68 | M04.02 | Combinations – selection with restrictions | Standard | 150 |
| 69 | M05.02 | Binomial theorem – middle term | Standard | 150 |
| 70 | M06.03 | Sequences – insertion of means | Standard | 220 |
| 71 | M07.03 | Limits – standard algebraic limits | Foundation | 150 |
| 72 | M08.03 | Integration – partial fractions | Standard | 150 |
| 73 | M09.02 | Differential equations – separable equations | Standard | 150 |
| 74 | M10.02 | Coordinate geometry – locus from a geometric constraint | Standard | 150 |
| 75 | M11.02 | 3D geometry – direction cosines and ratios | Challenging | 220 |

# PART A : PHYSICS

Detailed solutions

## Q.1

Dimensional consistency • Foundation • 65 s **Answer: C**

A resistive force is written $F = av + bv^2$, where $v$ is speed. What are the dimensions of $a/b$?

(A) $\mathrm{M\,L^{-1}}$ (B) $\mathrm{L^2\,T^{-2}}$ (C) $\mathrm{L\,T^{-1}}$ (D) $\mathrm{T^{-1}}$

**Solution**

Each term must have the dimensions of force, $[F] = \mathrm{M\,L\,T^{-2}}$.

$$[a] = \frac{[F]}{[v]} = \mathrm{M\,T^{-1}}, \qquad [b] = \frac{[F]}{[v^2]} = \mathrm{M\,L^{-1}}.$$

$$\left[\frac{a}{b}\right] = \frac{\mathrm{M\,T^{-1}}}{\mathrm{M\,L^{-1}}} = \mathrm{L\,T^{-1}}.$$

This is just the dimension of speed, as it must be, since $av$ and $bv^2$ are equal when $v = a/b$. Option (A) is $[b]$ alone, not the ratio.

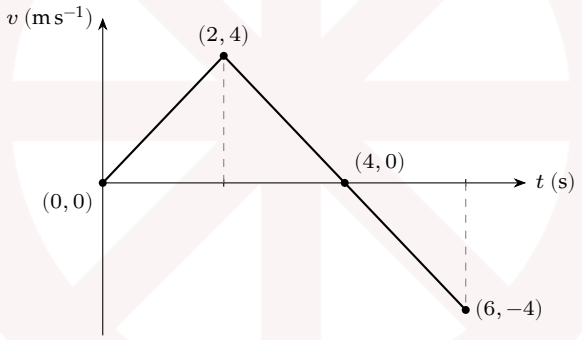
**Answer: (C)**

**Common mistake:** Reporting the dimensions of $b$ alone ($\mathrm{M\,L^{-1}}$) or inverting the ratio to $b/a$.

## Q.2

Displacement-time and velocity-time graphs • Foundation • 115 s **Answer: B**

The plotted velocity of a particle joins $(0,0)$, $(2,4)$, $(4,0)$, $(6,-4)$, with $t$ in s and $v$ in $\mathrm{m\,s^{-1}}$. Its distance travelled divided by the magnitude of displacement in 6 s is




(A) 1 (B) 3 (C) 4 (D) 2

**Solution**

Area under the $v$–$t$ graph gives displacement.

- 0 to 4 s: triangle above the axis, area $\frac{1}{2}(4)(4) = 8$ m.
- 4 to 6 s: triangle below the axis, area $\frac{1}{2}(2)(-4) = -4$ m.

$$\text{displacement} = 8 - 4 = 4\ \text{m}, \qquad \text{distance} = 8 + 4 = 12\ \text{m}.$$

$$\frac{\text{distance}}{|\text{displacement}|} = \frac{12}{4} = 3.$$

Option (A), 1, comes from using the signed area for the distance as well.

**Answer: (B)**

**Common mistake:** Using the signed (net) area for distance, which makes distance equal displacement and gives 1.

**Q.3** Free-body diagrams and connected masses • Foundation • 65 s **Answer: D**

Two blocks of masses 2 kg and 3 kg touch on a smooth horizontal floor. A horizontal 10 N force pushes the 2 kg block toward the 3 kg block. The contact force is

(A) 4 N (B) 10 N (C) 15 N (D) 6 N

**Solution**

Both blocks move together:

$$a = \frac{10}{2+3} = 2\ \mathrm{m\,s^{-2}}.$$

The only horizontal force on the 3 kg block is the contact force $N$:

$$N = 3 \times 2 = 6\ \mathrm{N}.$$

Option (B), 10 N, assumes the full applied force is passed on; part of it (4 N) is used to accelerate the 2 kg block. Option (A), 4 N, is that part, not the contact force.

**Answer: (D)**

**Common mistake:** Taking the contact force equal to the applied 10 N, or using the pushed block's mass ($2 \times 2 = 4$ N).

**Q.4** Work from force-position graph • Standard • 115 s **Answer: A**

A 1 kg cart starts at rest at $x = 0$. Its net horizontal force varies linearly from 4 N at $x = 0$ to $-2$ N at $x = 3$ m. Its speed at $x = 3$ m is

(A) $\sqrt{6}\ \mathrm{m\,s^{-1}}$ (B) 0 (C) $3\ \mathrm{m\,s^{-1}}$ (D) $\sqrt{12}\ \mathrm{m\,s^{-1}}$

**Solution**

The work done is the signed area under the $F$–$x$ line. For a straight line it equals the average force times the displacement:

$$W = \frac{4 + (-2)}{2} \times 3 = 3\ \mathrm{J}.$$

(Check: the force is zero at $x = 2$ m; the area is $+4$ J from 0 to 2 and $-1$ J from 2 to 3.)

Work–energy theorem:

$$\tfrac{1}{2}(1)v^2 = 3 \ \Rightarrow\ v = \sqrt{6}\ \mathrm{m\,s^{-1}}.$$

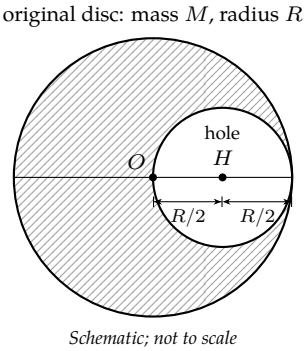
The cart does not stop, because the 4 J gained earlier exceeds the 1 J lost, so (B) is wrong. Option (D) would need 6 J of net work, more than even the positive area alone.

**Answer: (A)**

**Common mistake:** Treating the area beyond $x = 2$ m as positive work, or assuming the cart stops once the force reverses.

**Q.5** Moment of inertia and axes theorems • Challenging • 185 s **Answer: A**

A uniform disc of original mass $M$ and radius $R$ has an internally tangent circular hole of radius $R/2$ removed, with its centre $H$ at distance $R/2$ to the right of the original centre $O$. The remaining lamina rotates about an axis through its own centre of mass perpendicular to its plane. Its moment of inertia is




(A) $\dfrac{37MR^2}{96}$ (B) $\dfrac{3MR^2}{8}$

(C) $\dfrac{13MR^2}{32}$ (D) $\dfrac{5MR^2}{12}$

**Solution**

**Removed part.** Mass $\propto$ area, so $m = M/4$. Its inertia about $O$ (parallel axes):

$$I_{\text{hole},O} = \frac{1}{2}\cdot\frac{M}{4}\left(\frac{R}{2}\right)^2 + \frac{M}{4}\left(\frac{R}{2}\right)^2 = \frac{MR^2}{32} + \frac{MR^2}{16} = \frac{3MR^2}{32}.$$

**Remaining lamina about $O$:**

$$I_O = \frac{MR^2}{2} - \frac{3MR^2}{32} = \frac{13MR^2}{32}.$$

**Its centre of mass.** Mass $3M/4$ remains; $\frac{3M}{4}x_c + \frac{M}{4}\cdot\frac{R}{2} = 0$, so $x_c = -R/6$ (to the left of $O$).

**Shift to the CM axis:**

$$I_{\text{cm}} = I_O - \frac{3M}{4}\left(\frac{R}{6}\right)^2 = \frac{13MR^2}{32} - \frac{MR^2}{48} = \frac{39-2}{96}MR^2 = \frac{37MR^2}{96}.$$

Option (C), $13MR^2/32$, is the inertia about $O$, not about the remaining lamina's own centre of mass.

**Answer: (A)**

**Common mistake:** Stopping at $13MR^2/32$, the inertia about $O$, without shifting to the remaining lamina's centre of mass $R/6$ from $O$.

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## Q.6

Gravity versus altitude and depth • Foundation • 65 s **Answer: C**

At what altitude above a spherical planet of radius $R$ is gravitational acceleration one quarter of its surface value?

(A) $2R$ (B) $R/2$ (C) $R$ (D) $3R$

**Solution**

Outside the planet $g \propto 1/r^2$:

$$\frac{g(h)}{g_0} = \frac{R^2}{(R+h)^2} = \frac{1}{4} \Rightarrow R + h = 2R \Rightarrow h = R.$$

Option (A), $2R$, is the distance from the centre, not the altitude above the surface.

**Answer: (C)**

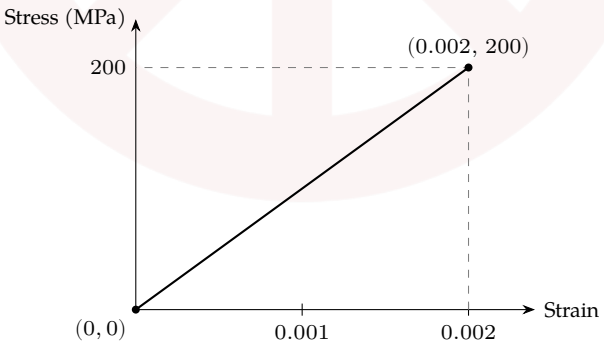
**Common mistake:** Reporting the distance from the centre $(2R)$ as the altitude.

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## Q.7

Stress-strain and elastic energy • Standard • 115 s **Answer: C**

A wire has the linear stress–strain graph through $(0, 0)$ and $(0.002,\ 200\ \text{MPa})$, and volume $2 \times 10^{-6}\ \text{m}^3$. It is first loaded to strain $0.001$ and then stretched quasistatically to $0.002$. The additional work in this second stage is




(A) 0.2 J (B) 0.1 J (C) 0.3 J (D) 0.4 J

**Solution**

Young's modulus from the slope:

$$Y = \frac{200 \times 10^6}{0.002} = 10^{11}\ \text{Pa}.$$

Elastic energy density at strain $\varepsilon$ is $u = \frac{1}{2}Y\varepsilon^2$. The increase from $\varepsilon = 0.001$ to $0.002$:

$$\Delta u = \tfrac{1}{2}(10^{11})\left[(0.002)^2 - (0.001)^2\right] = 1.5 \times 10^5\ \text{J m}^{-3}.$$

$$W = \Delta u \times V = 1.5 \times 10^5 \times 2 \times 10^{-6} = 0.3\ \text{J}.$$

Option (D), 0.4 J, is the total energy stored at strain $0.002$; option (B), 0.1 J, is the energy already stored at $0.001$.

**Answer: (C)**

**Common mistake:** Giving the total energy at strain 0.002 (0.4 J) instead of the increase from strain 0.001.

### Q.8

First-law sign convention • Foundation • 65 s **Answer: B**

During compression a gas releases 30 J of heat while 80 J of work is done on it. Its internal energy changes by

(A) $-50$ J (B) $+50$ J (C) $+110$ J (D) $-110$ J

**Solution**

Use $\Delta U = Q + W_{\text{on}}$, with $Q > 0$ for heat into the gas and $W_{\text{on}} > 0$ for work done on it.

$$Q = -30\ \text{J}, \qquad W_{\text{on}} = +80\ \text{J} \ \Rightarrow\ \Delta U = -30 + 80 = +50\ \text{J}.$$

The internal energy rises despite the heat loss. Option (D), $-110$ J, treats the 80 J as work done *by* the gas in $\Delta U = Q - W$; option (A) flips both signs.

**Answer: (B)**

**Common mistake:** Putting work done on the gas into $\Delta U = Q - W$ as if it were work done by the gas, giving $-110$ J.

### Q.9

Degrees of freedom • Foundation • 65 s **Answer: B**

Equal numbers of molecules of a monatomic and a rigid diatomic ideal gas are at the same temperature. Neglect vibration. Their total internal energies are in the ratio

(A) 5 : 3 (B) 3 : 5 (C) 3 : 7 (D) 1 : 1

**Solution**

$U = \frac{f}{2}NkT$. A monatomic molecule has $f = 3$ (translation only); a rigid diatomic has $f = 5$ (3 translational + 2 rotational).

$$\frac{U_{\text{mono}}}{U_{\text{dia}}} = \frac{3}{5}.$$

Option (A) is the inverted ratio; (C) would include the two vibrational modes that the question excludes.

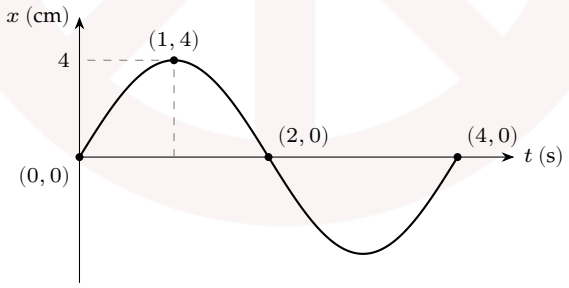
**Answer: (B)**

**Common mistake:** Writing the ratio upside down (5:3), or counting vibrational modes (f = 7) that the question tells you to neglect.

### Q.10

SHM phase and graphs • Standard • 185 s **Answer: D**

An SHM trace starts at $x = 0$ moving toward positive $x$. Its FIRST positive maximum, $+4$ cm, occurs at $t = 1$ s and its NEXT zero at $t = 2$ s. Between $t = 0$ and $t = 7/3$ s, the total distance travelled divided by the magnitude of displacement is




(A) 7 (B) 4 (C) 3 (D) 5

**Solution**

The first maximum is a quarter period after the start, so $T = 4$ s and

$$x = 4\sin\left(\frac{\pi t}{2}\right)\ \text{cm}.$$

At $t = 7/3$ s: $x = 4\sin\left(\frac{7\pi}{6}\right) = -2$ cm, so $|\text{displacement}| = 2$ cm.

Distance: $0 \to +4$ (4 cm), then $+4 \to 0 \to -2$ (6 cm), total 10 cm. There is no second turning point because the negative extreme is reached only at $t = 3$ s.

$$\frac{10}{2} = 5.$$

Option (C), 3, counts only the return leg from $+4$ to $-2$ and misses the first 4 cm.

**Answer: (D)**

**Common mistake:** Dropping the first 4 cm out to the positive extreme, or mis-evaluating $\sin(7\pi/6)$ as $+\frac{1}{2}$ so the displacement looks like $+2$ cm.

## Q.11

Coulomb force and equilibrium • Foundation • 65 s **Answer: D**

Positive charges $Q$ and $4Q$ are fixed 30 cm apart. A small positive test charge experiences zero net force between them at a distance from $Q$ of

(A) 15 cm (B) 6 cm (C) 20 cm (D) 10 cm

**Solution**

Let the distance from $Q$ be $x$ (cm). For zero net force,

$$\frac{Q}{x^2} = \frac{4Q}{(30-x)^2} \Rightarrow 30 - x = 2x \Rightarrow x = 10 \text{ cm}.$$

Option (B), 6 cm, splits the gap in the charge ratio $1:4$ instead of its square root $1:2$; option (C) is the distance from $4Q$.

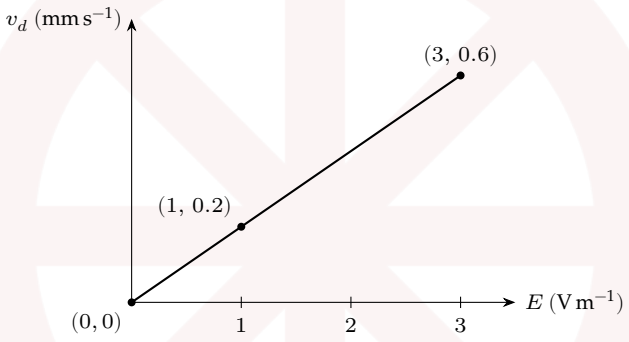
**Answer: (D)**

**Common mistake:** Dividing the 30 cm in the charge ratio 1:4 (giving 6 cm) instead of the square-root ratio 1:2.

## Q.12

Drift speed and mobility • Standard • 115 s **Answer: B**

A conductor has the measured drift-speed graph shown. A 2 m specimen of it carries current $I$ when connected to a 6 V supply. It is replaced by a 4 m specimen of the same material and temperature with half the cross-sectional area, connected to the same supply. The new current is




(A) $2I$ (B) $I/4$ (C) $I$ (D) $I/2$

**Solution**

The graph is a straight line through the origin, so the mobility $\mu = v_d/E = 0.2$ mm s$^{-1}$ per V m$^{-1}$ is constant.

- Field: $E = V/L$. Doubling $L$ at the same $V$ halves $E$, hence halves $v_d$.
- Current: $I = neAv_d$. Here $A$ is also halved.

$$\frac{I'}{I} = \frac{A/2}{A} \cdot \frac{v_d/2}{v_d} = \frac{1}{4} \Rightarrow I' = \frac{I}{4}.$$

(Same result from $R = \rho L/A$: $R$ becomes $4R$.) Option (D), $I/2$, accounts for the drift speed but forgets the halved area.

**Answer: (B)**

**Common mistake:** Halving the drift speed but forgetting that the halved cross-section also cuts the current (giving I/2).

## Q.13

Field of wire loop and solenoid • Standard • 115 s **Answer: C**

A long solenoid has $N$ turns over length $L$ and carries current $I$. Keeping $N$ and $I$ fixed, it is stretched to length $2L$ while its radius remains much smaller than $L$. A second coaxial long solenoid provides an opposing field equal to one quarter of the original first-solenoid field. The net interior field magnitude is

(A) three quarters of the original first-solenoid field
(B) one half of the original first-solenoid field
(C) one quarter of the original first-solenoid field
(D) zero

**Solution**

Original field $B_0 = \mu_0 NI/L$. Stretching to $2L$ halves the turns per unit length, so the first solenoid now gives $B_0/2$. The opposing field is $B_0/4$:

$$B_{\text{net}} = \frac{B_0}{2} - \frac{B_0}{4} = \frac{B_0}{4}.$$

Option (A), $\frac{3}{4}B_0$, forgets that stretching halves the first field.

**Answer: (C)**

**Common mistake:** Forgetting that stretching halves $n = N/L$ and subtracting $B_0/4$ from the original $B_0$ (giving $\frac{3}{4}B_0$).

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### Q.14

Flux change and Lenz law • Standard • 115 s **Answer: C**

A horizontal conducting ring of area $0.02\ \text{m}^2$ and resistance $2\ \Omega$ is viewed from above. Its area is increasing at $0.01\ \text{m}^2\,\text{s}^{-1}$ while an upward field of $0.4\ \text{T}$ is decreasing at $0.1\ \text{T}\,\text{s}^{-1}$. At that instant the induced current is

(A) 1 mA anticlockwise (B) 3 mA anticlockwise

(C) 1 mA clockwise (D) 3 mA clockwise

**Solution**

$$\frac{d\Phi}{dt} = B\frac{dA}{dt} + A\frac{dB}{dt} = (0.4)(0.01) + (0.02)(-0.1) = 0.004 - 0.002 = +0.002\ \text{Wb}\,\text{s}^{-1}.$$

The upward flux is **increasing**, so the induced current produces a downward field: clockwise when viewed from above.

$$I = \frac{0.002}{2} = 1\ \text{mA}.$$

Options (B)/(D), 3 mA, add the two terms instead of letting them compete; the anticlockwise options follow only the falling-field term.

**Answer: (C)**

**Common mistake:** Adding the two flux-change terms (giving 3 mA) or deciding the direction from the decreasing field alone.

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### Q.15

Displacement current • Standard • 115 s **Answer: D**

A parallel-plate capacitor of circular plate radius $a$ is charged at constant current $I$. Neglect fringing. For coaxial circular loops of radii $a/2$ and $2a$ lying in a plane between the plates (the loop of radius $2a$ extends beyond the plate edge), the ratio of magnetic fields $B(a/2)/B(2a)$ is

(A) 4 (B) 1/4 (C) 2 (D) 1

**Solution**

Displacement current is spread uniformly over the plate area.

- $r < a$: enclosed current $Ir^2/a^2$, so $B = \dfrac{\mu_0 I r}{2\pi a^2}$. At $r = a/2$: $B = \dfrac{\mu_0 I}{4\pi a}$.
- $r > a$: all of $I$ is enclosed, so $B = \dfrac{\mu_0 I}{2\pi r}$. At $r = 2a$: $B = \dfrac{\mu_0 I}{4\pi a}$.

$$\frac{B(a/2)}{B(2a)} = 1.$$

Option (A), 4, uses the full displacement current for the inner loop.

**Answer: (D)**

**Common mistake:** Using the full displacement current $I$ for the inner loop of radius $a/2$ (giving a ratio of 4).

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### Q.16

Spherical mirror image formation • Foundation • 65 s **Answer: A**

An object is placed 30 cm in front of a concave mirror of focal length 15 cm. Its image is

(A) real, inverted and equal in size (B) virtual, erect and enlarged

(C) at infinity (D) real, inverted and half-sized

**Solution**

The object is at $2f$, the centre of curvature. With $u = -30$ cm, $f = -15$ cm:

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{15} + \frac{1}{30} = -\frac{1}{30} \Rightarrow v = -30\ \text{cm}, \qquad m = -\frac{v}{u} = -1.$$

The image is real, inverted and the same size, at the centre of curvature. Option (C) is the case of an object at $F$; option (B) needs the object inside $F$.

**Answer: (A)**

**Common mistake:** Confusing the object at C (2f) with the object at F, and calling the image at infinity.

## Q.17

Photoelectric graphs and threshold • Standard • 115 s **Answer: A**

A photocathode gives stopping potentials 1 V and 3 V for photon energies 3 eV and 5 eV. It is next illuminated by 4 eV photons and a retarding potential of 0.5 V is applied. The largest electron kinetic energy at the collector is

(A) 1.5 eV (B) 2.5 eV (C) 0.5 eV (D) 2 eV

### Solution

From $eV_0 = h\nu - \phi$: $1 = 3 - \phi$ and $3 = 5 - \phi$, both giving $\phi = 2$ eV.

With 4 eV photons, $K_{\max} = 4 - 2 = 2$ eV at emission. The retarding 0.5 V removes 0.5 eV:

$$K_{\text{collector}} = 2 - 0.5 = 1.5 \text{ eV}.$$

Option (D), 2 eV, is the emission energy; (B) adds the retarding potential instead of subtracting it.

**Answer: (A)**

**Common mistake:** Reporting the emission kinetic energy (2 eV) and ignoring the energy lost against the retarding potential.

## Q.18

Diode IV and bias • Standard • 185 s **Answer: B**

Node $X$ is connected to $+10$ V through 2 kΩ and to ground through 3 kΩ. Two ideal diodes connect $X$ to reference supplies: $D_1$ has anode $X$ and cathode $+5$ V; $D_2$ has anode $+2$ V and cathode $X$. The steady current through $D_1$ is

(A) $\frac{5}{3}$ mA (B) $\frac{5}{6}$ mA

(C) $\frac{5}{2}$ mA (D) 0

### Solution

**Test both diodes off.** The divider gives $V_X = 10 \times \frac{3}{2+3} = 6 \text{ V} > 5$ V, so $D_1$ would be forward biased. The assumption fails.

**$D_1$ on.** $D_1$ clamps $V_X = 5$ V. $D_2$ has anode 2 V and cathode 5 V, so it is reverse biased (off). Consistent.

KCL at $X$:

$$I_{2\,\text{k}\Omega} = \frac{10 - 5}{2} = 2.5 \text{ mA}, \qquad I_{3\,\text{k}\Omega} = \frac{5}{3} \text{ mA},$$

$$I_{D_1} = \frac{5}{2} - \frac{5}{3} = \frac{5}{6} \text{ mA}.$$

Option (C) is the total current into $X$ and option (A) is the ground-branch current; neither is the diode current. Option (D) would need $V_X \le 5$ V with $D_1$ off, which the divider rules out.

**Answer: (B)**

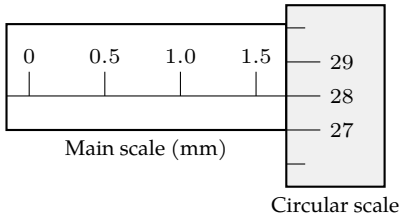
**Common mistake:** Taking the whole 2 kΩ current (5/2 mA) as the diode current, forgetting that 5/3 mA still flows to ground through 3 kΩ.

## Q.19

Vernier and screw gauge readings • Standard • 115 s **Answer: D**

The screw-gauge scale shown has pitch 0.5 mm and 50 circular divisions; its closed-jaw reading is $+0.03$ mm. This wire gives main reading 1.5 mm and circular reading 28. A second wire measured with the same gauge gives main reading 1.0 mm and circular reading 43. The ratio of corrected diameters, first to second, is

Pitch 0.5 mm; 50 divisions; zero error +0.03 mm




*Schematic; not to scale*

(A) $\frac{178}{143}$ (B) $\frac{4}{5}$

(C) $\frac{7}{5}$ (D) $\frac{5}{4}$

**Solution**

Least count $= \dfrac{0.5}{50} = 0.01$ mm. Zero error $= +0.03$ mm (subtract it).

- Wire 1: $1.5 + 28(0.01) = 1.78$ mm $\rightarrow$ $1.78 - 0.03 = 1.75$ mm.
- Wire 2: $1.0 + 43(0.01) = 1.43$ mm $\rightarrow$ $1.43 - 0.03 = 1.40$ mm.

$$\frac{d_1}{d_2} = \frac{1.75}{1.40} = \frac{5}{4}.$$

Option (A), 178/143, is the ratio of the uncorrected readings. An additive zero error does not cancel in a ratio, so it must be removed from each reading first. Option (B) is the inverted ratio.

**Answer: (D)**

**Common mistake:** Taking the ratio of the raw readings (178/143) on the belief that a common zero error cancels in a ratio.

## Q.20

Static versus kinetic friction • Challenging • 185 s **Answer: A**

A 2 kg block on a horizontal floor has $\mu_s = 0.5$ and $\mu_k = 0.3$. A force is applied at $37^\circ$ above horizontal ($\cos 37^\circ = 0.8$, $\sin 37^\circ = 0.6$); take $g = 10\ \mathrm{m\,s^{-2}}$. The force is raised slowly to the threshold for sliding and is then held at that magnitude. The acceleration just after sliding begins is

(A) $\dfrac{16}{11}\ \mathrm{m\,s^{-2}}$ (B) $\dfrac{20}{11}\ \mathrm{m\,s^{-2}}$

(C) $\dfrac{8}{11}\ \mathrm{m\,s^{-2}}$ (D) $0$

**Solution**

**Threshold.** $N = 20 - 0.6P$ and $0.8P = \mu_s N = 0.5(20 - 0.6P)$:

$$0.8P = 10 - 0.3P \ \Rightarrow\ P = \frac{100}{11}\ \mathrm{N}, \qquad N = 20 - \frac{60}{11} = \frac{160}{11}\ \mathrm{N}.$$

**Just after sliding.** $P$ and $N$ are unchanged, but friction drops to kinetic:

$$f_k = 0.3 \times \frac{160}{11} = \frac{48}{11}\ \mathrm{N}, \qquad P\cos 37^\circ = \frac{80}{11}\ \mathrm{N}.$$

$$a = \frac{80/11 - 48/11}{2} = \frac{32/11}{2} = \frac{16}{11}\ \mathrm{m\,s^{-2}}.$$

Option (D), 0, keeps the limiting static friction after motion starts.

**Answer: (A)**

**Common mistake:** Keeping limiting static friction after sliding starts (giving 0), or using $N = mg$ and ignoring the upward component $P\sin 37^\circ$.

## Q.21

Bohr transitions and spectral lines • Standard • 115 s **Answer: 3**

Hydrogen atoms are excited to $n = 5$ and all downward cascades occur in a large collection. An ideal filter removes every photon from a transition ending at $n = 1$ or $n = 2$. How many distinct spectral lines pass through the filter?

**Solution**

Transmitted lines must end at $n = 3$ or $n = 4$ (and start no higher than $n = 5$):

$$5 \to 4, \qquad 5 \to 3, \qquad 4 \to 3.$$

All three have different energies, so **3** distinct lines pass. (The total from $n = 5$ is $\binom{5}{2} = 10$; the 7 lines ending at $n = 1$ or 2 are removed.)

**Answer: 3**

**Common mistake:** Reporting all 10 lines from n = 5 (or the 6 lines from n = 4) before removing those that end at n = 1 or 2.

### Q.22

Significant figures • Standard • 115 s **Answer: 2**

A measured volume is calculated as $(12.11\ \text{cm}+0.3\ \text{cm}+1.024\ \text{cm})\times 2.0\ \text{cm}\times 1.00\ \text{cm}$. Use normal significant-figure rules, retaining guard digits until the end. Enter the number of significant figures in the final volume.

**Solution**

**Addition** is limited by the least number of decimal places, one (from 0.3): $12.11+0.3+1.024=13.434\rightarrow 13.4$ cm (3 s.f.; keep 13.434 as a guard value).

**Multiplication** is limited by the fewest significant figures, two (from 2.0):

$$13.434\times 2.0\times 1.00=26.868\ \rightarrow\ 27\ \text{cm}^3.$$

The final volume has **2** significant figures. Answering 3 applies the addition result's precision to the product.

**Answer: 2**

**Common mistake:** Stopping at the 3 significant figures of the sum (13.4 cm) and missing that the factor 2.0 cm limits the product to 2.

### Q.23

Relative motion in a plane • Foundation • 115 s **Answer: 20**

A boat moves at $5\ \text{m s}^{-1}$ relative to still water. A river flows east at $3\ \text{m s}^{-1}$. The boat is aimed so that it travels due north across a river 80 m wide; find the crossing time in seconds.

**Solution**

To cancel the drift, the boat's velocity relative to water needs a westward component of $3\ \text{m s}^{-1}$. Its northward component is

$$\sqrt{5^2-3^2}=4\ \text{m s}^{-1},\qquad t=\frac{80}{4}=20\ \text{s}.$$

Using the full $5\ \text{m s}^{-1}$ across the river gives 16 s, which is the shortest-time crossing, not the due-north one.

**Answer: 20**

**Common mistake:** Using the full 5 m/s as the cross-river speed (16 s), which is the minimum-time path, not the due-north path.

### Q.24

Conservative force and potential energy • Foundation • 115 s **Answer: 8**

A 2 kg particle has potential energy $U(x)=2x^2-8x+10$ J, with $x$ in metres. Released from rest at $x=0$, what is its maximum kinetic energy in joules?

**Solution**

Complete the square: $U=2(x-2)^2+2$ J, so $U_{\text{min}}=2$ J at $x=2$ m, which the particle reaches.

Total energy $E=U(0)+0=10$ J:

$$K_{\text{max}}=E-U_{\text{min}}=10-2=8\ \text{J}.$$

Answering 10 assumes $U$ falls to zero; it never goes below 2 J.

**Answer: 8**

**Common mistake:** Taking $K_{\text{max}}=U(0)=10$ J, as if the potential energy could fall to zero instead of to its minimum 2 J.

### Q.25

Angular momentum conservation • Standard • 185 s **Answer: 6**

A turntable of inertia $2\ \text{kg m}^2$ rotates at $6\ \text{rad s}^{-1}$ about a frictionless vertical axle. A 1 kg lump moving horizontally at $1.5\ \text{m s}^{-1}$ tangentially in the same rotational sense strikes and sticks at radius 1 m. It is then pulled slowly to radius 0.5 m without external torque about the axle. Find the final angular speed in $\text{rad s}^{-1}$.

**Solution**

Angular momentum about the axle is conserved throughout (no external axial torque), even though the sticking collision loses kinetic energy.

$$L=I\omega+mvr=2\times 6+1\times 1.5\times 1=13.5\ \text{kg m}^2\,\text{s}^{-1}.$$

Final inertia: $I_f=2+1\times(0.5)^2=2.25\ \text{kg m}^2$.

$$\omega_f=\frac{13.5}{2.25}=6\ \text{rad s}^{-1}.$$

Conserving kinetic energy in the collision, or leaving out the lump's own angular momentum $mvr$, gives a wrong value.

**Answer: 6**

**Common mistake:** Leaving out the lump's incoming angular momentum $mvr=1.5\ \text{kg m}^2\,\text{s}^{-1}$, or conserving kinetic energy in the sticking collision.

## PART B : CHEMISTRY — Detailed solutions

### Q.26

Mole concept – limiting reagent and yield • Foundation • 55 s **Answer: D**

Consider the following statements.

**Statement I:** For $2\,H_2 + O_2 \longrightarrow 2\,H_2O$, 3 mol $H_2$ mixed with 2 mol $O_2$ makes $H_2$ the limiting reagent.

**Statement II:** At 80% yield, this mixture produces 2.4 mol water.

Choose the correct option.

(A) I is correct; II is incorrect
(B) Both I and II are incorrect
(C) I is incorrect; II is correct
(D) Both I and II are correct

**Solution**

**Statement I: True.** By the balanced equation, 3 mol $H_2$ needs only $3/2 = 1.5$ mol $O_2$. Since 2 mol $O_2$ is available, $O_2$ is in excess and $H_2$ is limiting.

**Statement II: True.** The theoretical water yield is set by $H_2$: 3 mol $H_2$ gives 3 mol $H_2O$. At 80% yield,

$$n(H_2O) = 0.80 \times 3 = 2.4 \text{ mol}.$$

Hence option **(D)**. Option (A) is the trap for anyone who takes $O_2$ as limiting ($2$ mol $O_2 \rightarrow 4$ mol theoretical, 3.2 mol actual), which makes Statement II look false.

**Common mistake:** Taking $O_2$ as limiting because fewer moles are present.

### Q.27

Atomic structure – Bohr energy and spectrum • Standard • 95 s **Answer: C**

An electron in a hydrogen atom falls from $n = 4$ to $n = 2$. The same photon can ionise an excited hydrogen atom initially in which lowest principal quantum level?

(A) 2 (B) 4 (C) 3 (D) 1

**Solution**

Photon energy for $n = 4 \rightarrow 2$:

$$E = 13.6\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 13.6 \times \frac{3}{16} = 2.55 \text{ eV}.$$

Ionisation from level $n$ needs $13.6/n^2$ eV:

- $n = 2$: $3.4$ eV $> 2.55$ eV, not possible.
- $n = 3$: $13.6/9 \approx 1.51$ eV $< 2.55$ eV, possible.

So the lowest level that can be ionised is $n = 3$: option **(C)**. Option (A) is the tempting choice because the photon came from a transition ending at $n = 2$, but it carries less than the 3.4 eV needed to ionise from $n = 2$. Option (D) (ground state, 13.6 eV) is even further out of reach.

**Common mistake:** Using ionisation energy from $n = 2$ (3.4 eV) or picking the ground state.

### Q.28

Chemical bonding – Lewis structure and formal charge • Foundation • 55 s **Answer: D**

Consider the following statements.

**Statement I:** In an octet-obeying Lewis structure of CO, carbon has formal charge $-1$.

**Statement II:** In that same structure, oxygen has formal charge $+1$.

Choose the correct option.

(A) Both I and II are incorrect
(B) I is incorrect; II is correct
(C) I is correct; II is incorrect
(D) Both I and II are correct

**Solution**

The octet structure of CO has a triple bond and one lone pair on each atom. Formal charge = valence electrons − non-bonding electrons $-\frac{1}{2}$(bonding electrons).

**Statement I: True.** Carbon: $4 - 2 - \frac{6}{2} = -1$.

**Statement II: True.** Oxygen: $6 - 2 - \frac{6}{2} = +1$.

Hence option **(D)**. Options (A) and (B) tempt students who expect the more electronegative oxygen to carry the negative charge. That reasoning confuses formal charge with partial charge.

**Common mistake:** Putting the negative formal charge on oxygen because it is more electronegative, which confuses formal charge with bond polarity.

## Q.29

Thermodynamics – state functions and process work • Foundation • 95 s **Answer: C**

**Assertion (A):** An isolated ideal gas expanding freely into vacuum has $\Delta U = 0$.

**Reason (R):** During free expansion into vacuum the gas performs positive pressure–volume work on its surroundings.

Choose the correct option.

(A) Both A and R are true and R correctly explains A

(B) Both A and R are true but R does not correctly explain A

(C) A is true but R is false

(D) A is false but R is true

### Solution

**Assertion: True.** An isolated system exchanges no heat, so $q = 0$. Expansion against vacuum means $p_{\text{ext}} = 0$, so $w = -p_{\text{ext}}\Delta V = 0$. The first law gives $\Delta U = q + w = 0$.

**Reason: False.** With zero opposing pressure, the gas does no pressure–volume work during free expansion.

**Explanation:** A false Reason cannot explain the Assertion.

Hence option **(C)**: A is true but R is false. Option (A) or (B) tempts students who assume that any increase in volume means work is done. Work depends on the external pressure, and here it is zero.

**Common mistake:** Assuming the gas does work because its volume increases, and forgetting that $w = -p_{\text{ext}}\Delta V$ with $p_{\text{ext}} = 0$.

## Q.30

Solutions – concentration conversion • Challenging • 145 s **Answer: A**

A 40% by mass aqueous urea stock has density $1.20\ \text{g mL}^{-1}$. A student takes $50.0$ mL, adds $40.0$ g water, and evaporates $20.0$ g water without losing urea. The final density is $1.10\ \text{g mL}^{-1}$. With molar mass $60\ \text{g mol}^{-1}$, choose final molarity and molality.

(A) $5.50$ M and $7.14\ \text{mol kg}^{-1}$

(B) $4.40$ M and $5.00\ \text{mol kg}^{-1}$

(C) $7.14$ M and $5.50\ \text{mol kg}^{-1}$

(D) $5.50$ M and $5.00\ \text{mol kg}^{-1}$

### Solution

Stock portion: $50.0\ \text{mL} \times 1.20\ \text{g mL}^{-1} = 60.0$ g, containing $24.0$ g urea $(0.40\ \text{mol})$ and $36.0$ g water.

Final water: $36 + 40 - 20 = 56$ g. Final solution mass: $24 + 56 = 80$ g.

$$m = \frac{0.40\ \text{mol}}{0.056\ \text{kg}} = 7.14\ \text{mol kg}^{-1}$$

$$V = \frac{80\ \text{g}}{1.10\ \text{g mL}^{-1}} = 72.73\ \text{mL}, \qquad M = \frac{0.40\ \text{mol}}{0.07273\ \text{L}} = 5.50\ \text{M}$$

Hence option **(A)**. Option (C) has the right numbers with molarity and molality swapped. Option (D)'s $5.00\ \text{mol kg}^{-1}$ comes from dividing by the total solution mass (80 g) instead of the solvent mass.

**Common mistake:** Dividing by the total solution mass (80 g) instead of the 56 g of water for molality, or forgetting the 20 g of water lost by evaporation.

## Q.31

Chemical equilibrium – $K_p$–$K_c$ conversion • Foundation • 55 s **Answer: C**

Consider the following statements.

**Statement I:** For $\text{H}_2(\text{g}) + \text{I}_2(\text{g}) \rightleftharpoons 2\,\text{HI}(\text{g})$, $K_p$ and $K_c$ have equal numerical values under the usual ideal-gas convention.

**Statement II:** For $\text{N}_2\text{O}_4(\text{g}) \rightleftharpoons 2\,\text{NO}_2(\text{g})$, $K_p/K_c = RT$ under that convention.

Choose the correct option.

(A) Both I and II are incorrect

(B) I is incorrect; II is correct

(C) Both I and II are correct

(D) I is correct; II is incorrect

### Solution

For ideal gases, $K_p = K_c(RT)^{\Delta n_g}$, where $\Delta n_g$ = moles of gaseous products − moles of gaseous reactants.

**Statement I: True.** $\Delta n_g = 2 - (1 + 1) = 0$, so $K_p = K_c$.

**Statement II: True.** $\Delta n_g = 2 - 1 = 1$, so $K_p/K_c = (RT)^1 = RT$.

Hence option **(C)**. Option (D) tempts students who take $\Delta n = 2$ from the product coefficient, which would give $(RT)^2$.

**Common mistake:** Using $\Delta n = 2$ for $\text{N}_2\text{O}_4 \rightleftharpoons 2\,\text{NO}_2$ (counting coefficients, not the change).

## Q.32

Redox – oxidation number and back-titration • Standard • 145 s **Answer: B**

A solution initially contains 0.050 mol $Fe^{2+}$. It is treated with 0.010 mol $Cr_2O_7^{2-}$ in acid. The excess dichromate is then consumed by added $Fe^{2+}$. How many millimoles of additional $Fe^{2+}$ are required?

(A) 60 (B) 10 (C) 20 (D) 5

**Solution**

In acid, each $Cr_2O_7^{2-}$ gains $6e^-$ ($2\,Cr$: $+6 \to +3$), and each $Fe^{2+}$ loses $1e^-$:

$$Cr_2O_7^{2-} + 6\,Fe^{2+} + 14\,H^+ \longrightarrow 2\,Cr^{3+} + 6\,Fe^{3+} + 7\,H_2O$$

So 0.010 mol dichromate can oxidise $6 \times 0.010 = 0.060$ mol $Fe^{2+}$. Only 0.050 mol is present at first, so the remaining capacity is

$$0.060 - 0.050 = 0.010\ \text{mol} = 10\ \text{mmol}\ Fe^{2+}.$$

Hence option **(B)**. Option (A) is the full capacity of the dichromate and ignores the $Fe^{2+}$ already present. Options (C) and (D) come from ratios other than 6:1.

**Common mistake:** Using a 1:1 or 1:5 $Fe^{2+}$:$Cr_2O_7^{2-}$ ratio instead of 6:1.

## Q.33

Chemical kinetics – rate law from data • Standard • 55 s **Answer: A**

Consider the following statements.

**Statement I:** If doubling [A] at fixed [B] multiplies the initial rate by four, the order in A is two.

**Statement II:** If doubling [B] at fixed [A] leaves the rate unchanged, B must be absent from the balanced chemical equation.

Choose the correct option.

(A) I is correct; II is incorrect (B) Both I and II are correct

(C) Both I and II are incorrect (D) I is incorrect; II is correct

**Solution**

Take rate $= k[A]^a[B]^b$.

**Statement I: True.** $2^a = 4$ gives $a = 2$.

**Statement II: False.** $2^b = 1$ gives $b = 0$, so the observed order in B is zero. A reactant can be zero order under the experimental conditions (for example, when it reacts after the rate-determining step) and still appear in the balanced equation.

Hence option **(A)**. Option (B) tempts students who think a zero-order reactant cannot take part in the reaction.

**Common mistake:** Assuming a zero-order reactant must be absent from the balanced equation, which confuses the rate law with the stoichiometry.

## Q.34

Periodicity – atomic and ionic radii • Foundation • 95 s **Answer: B**

Consider the following statements.

**Statement I:** Among $O^{2-}$, $F^-$, $Na^+$ and $Mg^{2+}$, $Mg^{2+}$ has the smallest ionic radius.

**Statement II:** The same four ions all contain ten electrons.

Choose the correct option.

(A) Both I and II are incorrect (B) Both I and II are correct

(C) I is incorrect; II is correct (D) I is correct; II is incorrect

**Solution**

**Statement I: True.** For an isoelectronic series, a larger nuclear charge pulls the same 10 electrons in more tightly. $Mg^{2+}$ ($Z = 12$) is the smallest: $O^{2-} > F^- > Na^+ > Mg^{2+}$.

**Statement II: True.** Electron counts: $O^{2-}$ $8 + 2$, $F^-$ $9 + 1$, $Na^+$ $11 - 1$, $Mg^{2+}$ $12 - 2$. All have 10 electrons, so the ions are isoelectronic.

Hence option **(B)**. Option (D) tempts students who do not count the electrons and assume the ions differ.

**Common mistake:** Missing that the four ions are isoelectronic, and so picking an ion of a larger atom as the smallest instead of ranking by nuclear charge.

**Q.35** p-Block – group 13 trends and first-element anomaly • Foundation • 95 s **Answer: B**

Consider the following statements.

**Statement I:** The stability of the $+1$ oxidation state generally increases from aluminium to thallium.

**Statement II:** Boron commonly forms a simple $B^{3+}$ ion in aqueous solution because of its small size.

Choose the correct option.

(A) I is incorrect; II is correct
(B) I is correct; II is incorrect
(C) Both I and II are incorrect
(D) Both I and II are correct

**Solution**

**Statement I: True.** The inert-pair effect increasingly stabilises the lower oxidation state down group 13. $Tl^{+}$ is more stable than $Tl^{3+}$.

**Statement II: False.** Boron's small size and very high ionisation enthalpies $(I_1 + I_2 + I_3)$ make a free $B^{3+}$ ion energetically unfavourable. Boron forms predominantly covalent compounds.

Hence option **(B)**. Option (D) is the trap: small size works *against* simple cation formation, because the charge density would be extreme.

**Common mistake:** Thinking that boron's small size favours a simple $B^{3+}$ ion, when small size and high ionisation enthalpy are why boron bonds covalently.

**Q.36** d-Block – transition configurations and oxidation states • Foundation • 55 s **Answer: D**

Which ion has a $3d^5$ configuration? Atomic numbers: Cr 24, Mn 25, Fe 26, Co 27.

(A) $Fe^{2+}$ (B) $Cr^{3+}$ (C) $Co^{2+}$ (D) $Mn^{2+}$

**Solution**

Neutral Mn is $[Ar]\,3d^5 4s^2$. Removing the two $4s$ electrons gives $Mn^{2+}$: $[Ar]\,3d^5$.

The others: $Cr^{3+}$ $(3d^3)$, $Fe^{2+}$ $(3d^6)$, $Co^{2+}$ $(3d^7)$.

Hence option **(D)**. Option (A) is the trap: $Fe^{3+}$, not $Fe^{2+}$, is $3d^5$. Option (B) tempts students who remember that neutral Cr is $3d^5 4s^1$.

**Common mistake:** Removing $3d$ electrons before $4s$, or mixing up $Fe^{2+}$ $(3d^6)$ and $Fe^{3+}$ $(3d^5)$.

**Q.37** Coordination compounds – nomenclature and formula • Foundation • 95 s **Answer: C**

Match List I with List II.

| List I | List II |
|---|---|
| P. $K_4[Fe(CN)_6]$ | 1. Diamminedichloridoplatinum(II) |
| Q. $[Co(NH_3)_6]Cl_3$ | 2. Potassium hexacyanidoferrate(II) |
| R. $[Pt(NH_3)_2Cl_2]$ | 3. Hexaamminecobalt(III) chloride |

Choose the correct option.

(A) P-1, Q-2, R-3 (B) P-2, Q-1, R-3 (C) P-2, Q-3, R-1 (D) P-1, Q-3, R-2

**Solution**

- **P:** six $CN^{-}$ ligands contribute $-6$ and four $K^{+}$ contribute $+4$, so Fe is $+2$ in an anionic complex: potassium hexacyanidoferrate(II), **P-2**.
- **Q:** three $Cl^{-}$ counter-ions balance a $+3$ complex cation; $NH_3$ is neutral, so Co is $+3$: hexaamminecobalt(III) chloride, **Q-3**.
- **R:** two neutral $NH_3$ and two $Cl^{-}$ in a neutral complex put Pt at $+2$: diamminedichloridoplatinum(II), **R-1**.

Hence option **(C)**. Option (B) swaps Q and R, but only one formula contains platinum, so Q cannot be the platinum complex.

**Common mistake:** Treating $NH_3$ as a charged ligand, or ignoring the counter-ions, which gives wrong metal oxidation states.

**Q.38** Purification – crystallisation • Foundation • 55 s **Answer: A**

**Assertion (A):** A small amount of hot solvent is usually preferable to a large excess for recrystallising a solid with strongly temperature-dependent solubility.

**Reason (R):** Using excess solvent can leave more product dissolved in the cold mother liquor.

Choose the correct option.

(A) Both A and R are true and R correctly explains A
(B) Both A and R are true but R does not correctly explain A

(C) A is true but R is false

(D) A is false but R is true

**Solution**

**Assertion: True.** The hot solvent should be just enough to dissolve the crude solid.

**Reason: True.** At the cold temperature, the solvent still holds a saturated amount of product. More solvent means more product remains dissolved in the mother liquor and is lost.

**Explanation: R correctly explains A.** The lower recovery with excess solvent is exactly why a small, sufficient volume of hot solvent is preferred.

Hence option **(A)**. Option (B) tempts students who see R as a side fact. In fact R is the direct reason for the practice in A.

**Common mistake:** Thinking extra solvent gives purer crystals at no cost to yield.

## Q.39

Organic chemistry – IUPAC nomenclature and functional groups • Foundation • 95 s **Answer: B**

Match List I with List II.

| List I | List II |
|---|---|
| P. $CH_3CH_2CHO$ | 1. Carboxylic acid; propanoic acid |
| Q. $CH_3COCH_3$ | 2. Aldehyde; propanal |
| R. $CH_3CH_2COOH$ | 3. Ketone; propan-2-one |

Choose the correct option.

(A) P-2, Q-1, R-3 (B) P-2, Q-3, R-1 (C) P-1, Q-3, R-2 (D) P-1, Q-2, R-3

**Solution**

- **P:** $CH_3CH_2CHO$ has a terminal CHO group, so it is an aldehyde: propanal, **P-2**.
- **Q:** $CH_3COCH_3$ has the carbonyl carbon between two carbons, so it is a ketone: propan-2-one, **Q-3**.
- **R:** $CH_3CH_2COOH$ has a COOH group, so it is a carboxylic acid: propanoic acid, **R-1**.

Hence option **(B)**. Option (A) matches Q with the acid. A ketone has no OH on the carbonyl carbon.

**Common mistake:** Confusing CHO (aldehyde) with an in-chain C=O (ketone), because all three are $C_3$ carbonyl compounds.

## Q.40

Hydrocarbons – ethane conformations • Challenging • 145 s **Answer: A**

For ethane, let the torsional potential be $V(\theta) = \frac{V_0}{2}(1+\cos 3\theta)$, with $\theta$ measured by rotation of one $CH_3$ group relative to the other about the C–C bond, and $\theta = 0^\circ$ fully eclipsed. Over $0^\circ < \theta < 240^\circ$, how many stationary conformations occur, and how many are minima?

(A) 3 and 2 (B) 4 and 2 (C) 3 and 1 (D) 4 and 3

**Solution**

$$\frac{dV}{d\theta} = -\frac{3V_0}{2}\sin 3\theta = 0 \;\Rightarrow\; 3\theta = k\cdot 180^\circ \;\Rightarrow\; \theta = 0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, \ldots$$

In the open interval $0^\circ < \theta < 240^\circ$ only $\theta = 60^\circ, 120^\circ, 180^\circ$ qualify, so there are **3** stationary conformations.

- $60^\circ$ and $180^\circ$: $\cos 3\theta = -1$, so $V = 0$. These are staggered minima.
- $120^\circ$: $\cos 3\theta = +1$, so $V = V_0$. This is an eclipsed maximum.

So there are **2** minima. Hence option **(A)**. Option (B) wrongly counts $240^\circ$, which lies outside the open interval. Option (D) also counts every stationary point as a minimum.

**Common mistake:** Counting $\theta = 240^\circ$ (outside the open interval) or treating every stationary point as a minimum.

## Q.41

Haloalkanes – C–X bond and S_N2 reactivity • Foundation • 55 s **Answer: D**

For methyl halides reacting by $S_N2$ with the same nucleophile in the same solvent, which generally reacts fastest?

(A) $CH_3Br$ (B) $CH_3Cl$ (C) $CH_3F$ (D) $CH_3I$

**Solution**

The carbon skeleton and steric environment are the same for all four, so the rate depends on the leaving group. The C–I bond is the weakest C–X bond, and $I^-$ (the weakest base) is the best leaving group. So the order is $CH_3I > CH_3Br > CH_3Cl > CH_3F$.

Hence option **(D)**. Option (C) tempts students who reason from bond polarity. The C–F bond is the most polar, but it is also the strongest, and $F^-$ is a poor leaving group.

**Common mistake:** Choosing $CH_3F$ because the C–F bond is the most polar, and ignoring bond strength and leaving-group ability.

## Q.42

Alcohols – classification and Lucas test • Foundation • 95 s **Answer: D**

Which alcohol gives turbidity most rapidly with Lucas reagent at room temperature?

(A) Propan-1-ol
(B) Propan-2-ol
(C) Ethanol
(D) 2-Methylpropan-2-ol

**Solution**

Lucas reagent (conc. HCl + anhydrous $ZnCl_2$) converts alcohols by an $S_N1$-type path to insoluble alkyl chlorides, which cause turbidity. The rate follows carbocation stability: tertiary > secondary > primary. 2-Methylpropan-2-ol is the only tertiary alcohol here and gives turbidity almost at once.

Hence option **(D)**. Propan-2-ol (B) is secondary and reacts only within minutes. Propan-1-ol and ethanol are primary and show no turbidity at room temperature.

**Common mistake:** Assuming primary alcohols react fastest, as they do in oxidation, instead of ranking by carbocation stability.

## Q.43

Amines – preparation • Standard • 95 s **Answer: A**

Match List I with List II.

| List I | List II |
|---|---|
| P. $CH_3CONH_2$ with $Br_2$/NaOH | 1. Ethylamine |
| Q. $CH_3CN$ with $LiAlH_4$, then water | 2. Aniline |
| R. $C_6H_5NO_2$ with Sn/HCl, then base | 3. Methylamine |

Choose the correct option.

(A) P-3, Q-1, R-2
(B) P-2, Q-1, R-3
(C) P-1, Q-2, R-3
(D) P-1, Q-3, R-2

**Solution**

- **P:** Hofmann bromamide degradation removes the carbonyl carbon: $CH_3CONH_2 \longrightarrow CH_3NH_2$, methylamine, **P-3**.
- **Q:** Nitrile reduction keeps the nitrile carbon: $CH_3C{\equiv}N \longrightarrow CH_3CH_2NH_2$, ethylamine, **Q-1**.
- **R:** Sn/HCl reduces the nitro group to the anilinium salt, and base frees aniline $C_6H_5NH_2$, **R-2**.

Hence option **(A)**. Option (D) assumes Hofmann degradation keeps both carbons (P-1). The key feature of this reaction is the loss of one carbon.

**Common mistake:** Forgetting that Hofmann degradation loses one carbon while nitrile reduction keeps every carbon, which swaps methylamine and ethylamine.

## Q.44

Biomolecules – sucrose, lactose and maltose • Foundation • 95 s **Answer: A**

Which disaccharide gives only D-glucose on complete hydrolysis?

(A) Maltose
(B) Lactose
(C) Sucrose
(D) All three

**Solution**

- Maltose → D-glucose + D-glucose (α-1,4 link).
- Lactose → D-glucose + D-galactose.
- Sucrose → D-glucose + D-fructose.

Only maltose gives glucose alone. Hence option **(A)**. Option (C) tempts students who remember that sucrose contains glucose, but forget its fructose unit. Option (D) fails for the same reason.

**Common mistake:** Choosing sucrose or lactose because each contains a glucose unit, forgetting the fructose or galactose partner.

## Q.45

Practical chemistry – Mohr salt and potash alum preparation • Standard • 95 s **Answer: C**

Consider the following statements.

**Statement I:** Crystallising Mohr salt from 0.10 mol $FeSO_4$ and 0.08 mol $(NH_4)_2SO_4$ has a maximum theoretical yield of 0.08 mol $(NH_4)_2Fe(SO_4)_2 \cdot 6\,H_2O$, if all required water is available.

**Statement II:** Crystallising potash alum from 0.10 mol $K_2SO_4$ and 0.08 mol $Al_2(SO_4)_3$ has a maximum theoretical yield of 0.08 mol $KAl(SO_4)_2 \cdot 12\,H_2O$.

Choose the correct option. Assume sufficient water and no side reactions in both cases.

(A) I is incorrect; II is correct
(B) Both I and II are correct
(C) I is correct; II is incorrect
(D) Both I and II are incorrect

**Solution**

**Statement I: True.** $FeSO_4 + (NH_4)_2SO_4 + 6\,H_2O \longrightarrow (NH_4)_2Fe(SO_4)_2 \cdot 6\,H_2O$. The ratio is 1:1:1, so 0.08 mol $(NH_4)_2SO_4$ is limiting and gives at most 0.08 mol Mohr salt.

**Statement II: False.** $K_2SO_4 + Al_2(SO_4)_3 + 24\,H_2O \longrightarrow 2\,KAl(SO_4)_2 \cdot 12\,H_2O$. One formula unit of each sulfate gives **two** units of $KAl(SO_4)_2 \cdot 12\,H_2O$. $Al_2(SO_4)_3$ (0.08 mol) is limiting, so the maximum yield is $2 \times 0.08 = 0.16$ mol, not 0.08 mol.

Hence option **(C)**. Option (B) is the trap: it treats the double formula $K_2SO_4 \cdot Al_2(SO_4)_3 \cdot 24\,H_2O$ as one unit of $KAl(SO_4)_2 \cdot 12\,H_2O$.

**Common mistake:** Taking 1 mol $K_2SO_4 \cdot Al_2(SO_4)_3 \cdot 24\,H_2O$ as 1 mol of $KAl(SO_4)_2 \cdot 12\,H_2O$.

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## Q.46

Mole concept – empirical and molecular formula • Foundation • 95 s **Answer: 6**

An organic compound has empirical formula $CH_2O$ and molar mass $180\ g\ mol^{-1}$. How many oxygen atoms occur in one molecule? Use C = 12, H = 1, O = 16.

**Solution**

Empirical-formula mass $= 12 + 2(1) + 16 = 30$. Multiplier $n = 180/30 = 6$. The molecular formula is $C_6H_{12}O_6$, so one molecule contains **6** oxygen atoms. Answering 1 reads the empirical formula as the molecular formula.

**Common mistake:** Answering 1 from the empirical formula without applying the multiplier $180/30 = 6$.

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## Q.47

Atomic structure – quantum numbers and orbital capacity • Foundation • 95 s **Answer: 10**

How many electrons can occupy all orbitals having $n = 4$ and $l = 2$?

**Solution**

$n = 4, l = 2$ is the $4d$ subshell. It has $m_l = -2, -1, 0, +1, +2$, which is 5 orbitals. Each orbital holds 2 electrons, so the capacity is $5 \times 2 = \mathbf{10}$. Answering 5 counts orbitals, not electrons. Answering 32 $(2n^2)$ counts the whole $n = 4$ shell.

**Common mistake:** Answering 5 (orbitals, not electrons) or 32 (the whole $n = 4$ shell).

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## Q.48

Chemical bonding – Born–Haber lattice enthalpy cycle • Standard • 145 s **Answer: 1250**

For $MX_2(s)$, $\Delta H_f^\circ = -600\ kJ\ mol^{-1}$ and lattice dissociation enthalpy is $+2100\ kJ\ mol^{-1}$. Atomising M (s) costs $150\ kJ\ mol^{-1}$, breaking $X_2(g)$ costs $200\ kJ\ mol^{-1}$, and each X atom gains an electron with $\Delta H = -300\ kJ\ mol^{-1}$. If the first ionisation enthalpy of M is $500\ kJ\ mol^{-1}$, find its second ionisation enthalpy in $kJ\ mol^{-1}$.

**Solution**

The cycle for $M(s) + X_2(g) \longrightarrow MX_2(s)$: atomise M, break $X_2$ into two X atoms, ionise M twice, add one electron to **each** of the two X atoms, then form the lattice (the reverse of lattice dissociation, so $-2100$):

$$-600 = 150 + 200 + 500 + IE_2 - 600 - 2100$$

$$-600 = IE_2 - 1850 \;\Rightarrow\; IE_2 = 1250\ kJ\ mol^{-1}.$$

Counting the electron gain only once gives 950. Using $+2100$ for lattice formation gives an impossible negative $IE_2$.

**Common mistake:** Counting electron gain once instead of twice for 2X, or getting the sign of the lattice term wrong.

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## Q.49

Thermodynamics – Hess's law and formation enthalpy • Foundation • 95 s **Answer: 891**

Given $\Delta H_f^\circ$ of $CO_2(g)$, $H_2O(l)$ and $CH_4(g)$ as $-394$, $-286$ and $-75\ kJ\ mol^{-1}$, find the magnitude of methane combustion enthalpy in $kJ\ mol^{-1}$.

**Solution**

$$CH_4(g) + 2\,O_2(g) \longrightarrow CO_2(g) + 2\,H_2O(l)$$

$$\Delta_c H^\circ = [-394 + 2(-286)] - [-75 + 0] = -966 + 75 = -891\ kJ\ mol^{-1}$$

$O_2$ is an element in its standard state, so its formation enthalpy is 0. The magnitude is **891**. Forgetting the coefficient 2 on water gives 605.

**Common mistake:** Forgetting the coefficient 2 on $H_2O$ (giving 605), or subtracting the reactant term with the wrong sign (giving 1041).

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**Q.50** Solutions – Raoult's law and vapour–liquid split • Challenging • 145 s **Answer: 20**

A binary mixture contains an overall mole fraction 0.36 of A and forms coexisting ideal liquid and vapour phases. At the equilibrium temperature, $p_{\mathrm{A}}^{\circ} = 350$ kPa and $p_{\mathrm{B}}^{\circ} = 100$ kPa, and total pressure is 175 kPa. What percentage of the total moles is in the vapour phase?

**Solution**

Raoult's law for the liquid:

$$175 = 350x_{\mathrm{A}} + 100(1 - x_{\mathrm{A}}) \;\Rightarrow\; x_{\mathrm{A}} = 0.30$$

Dalton's law for the vapour: $y_{\mathrm{A}} = \dfrac{350 \times 0.30}{175} = 0.60.$

Let $\beta$ be the fraction of all moles in the vapour. A mole balance on A gives

$$0.36 = 0.30(1 - \beta) + 0.60\beta \;\Rightarrow\; 0.30\beta = 0.06 \;\Rightarrow\; \beta = 0.20,$$

so **20%** of the moles are in the vapour. Putting 0.36 directly into Raoult's law gives a total pressure (190 kPa) that does not match 175 kPa. The overall composition is not the liquid composition.

**Common mistake:** Using the overall mole fraction 0.36 directly in Raoult's law.

## PART C : MATHEMATICS

Detailed solutions

### Q.51

Sets – set operations and cardinality • Foundation • 80 s **Answer: C**

In a class of 60, 35 study French and 28 study German; 10 study neither. How many study exactly one of these languages?

(A) 38 (B) 39 (C) 37 (D) 36

**Solution**

Students studying at least one language: $n(F \cup G) = 60 - 10 = 50$.

$$n(F \cap G) = n(F) + n(G) - n(F \cup G) = 35 + 28 - 50 = 13.$$

Exactly one language: $n(F \cup G) - n(F \cap G) = 50 - 13 = 37$.

Subtracting the intersection twice from the total $35 + 28 = 63$ also gives $63 - 2(13) = 37$, which confirms the count. The value 50 (the union) counts the 13 students who take both languages, so it is not the answer.

**Answer: (C)**

**Common mistake:** Subtracting "neither" twice, or reporting the intersection.

### Q.52

Complex numbers – modulus and argument • Foundation • 80 s **Answer: A**

If $z = \dfrac{1 + i\sqrt{3}}{\sqrt{3} - i}$, find the principal argument of $z$.

(A) $\dfrac{\pi}{2}$ (B) $\dfrac{2\pi}{3}$

(C) $-\dfrac{\pi}{2}$ (D) $\dfrac{\pi}{6}$

**Solution**

$\arg(1 + i\sqrt{3}) = \dfrac{\pi}{3}$ and $\arg(\sqrt{3} - i) = -\dfrac{\pi}{6}$. For a quotient the arguments subtract:

$$\arg z = \frac{\pi}{3} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{2},$$

which already lies in $(-\pi, \pi]$. Check: $z = \dfrac{(1 + i\sqrt{3})(\sqrt{3} + i)}{4} = \dfrac{4i}{4} = i$.

Adding the arguments instead of subtracting gives $\pi/6$, and missing the sign of the denominator's argument gives $\pi/6$ as well. Both mistakes lead to option (D).

**Answer: (A)**

**Common mistake:** Adding the arguments of the numerator and denominator (or dropping the minus sign of $\arg(\sqrt{3} - i)$), which gives $\pi/6$.

### Q.53

Matrices – types and operations • Foundation • 80 s **Answer: D**

$A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$. Find the sum of the off-diagonal entries of $A^2$.

(A) 7 (B) 6 (C) 4 (D) 5

**Solution**

$$A^2 = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 2 \cdot 1 + 1 \cdot 3 \\ 0 & 9 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 0 & 9 \end{pmatrix}.$$

The off-diagonal entries are 5 and 0, so their sum is 5.

Squaring each entry gives $\begin{pmatrix} 4 & 1 \\ 0 & 9 \end{pmatrix}$ and a sum of 1, which is not an option. This is a common slip: matrix squaring is not entrywise.

**Answer: (D)**

**Common mistake:** Squaring each entry of $A$ instead of multiplying $A \cdot A$, which misses the $2 \cdot 1 + 1 \cdot 3$ cross terms.

**Q.54** Permutations – constrained arrangements • Standard • 150 s **Answer: A**

Six distinct books, including a physics and a mathematics book, are arranged on a shelf. How many arrangements have exactly two books between these two specified books?

(A) 144 (B) 240 (C) 288 (D) 72

**Solution**

Positions with exactly two places between them: $(1,4),(2,5),(3,6)$, which gives 3 position pairs.
The two specified books can swap: 2 ways. The other four books fill the remaining places in $4! = 24$ ways.

$$3 \times 2 \times 24 = 144.$$

Forgetting that the two books can swap gives 72 (D), and counting the position pairs twice gives 288 (C).
**Answer: (A)**

**Common mistake:** Forgetting the 2! order of the two specified books (giving 72), or counting $(1,4)$ and $(4,1)$ as separate position pairs as well as swapping (giving 288).

**Q.55** Binomial theorem – general term • Standard • 220 s **Answer: D**

In $\left(x^2 + \frac{1}{x}\right)^n$, $n$ is a positive integer. The ratio of the coefficient of $x^6$ to that of $x^3$ is $\frac{3}{2}$, and both terms occur. Find $n$.

(A) 12 (B) 18 (C) 6 (D) 24

**Solution**

$$T_{r+1} = \binom{n}{r}(x^2)^{n-r}x^{-r} = \binom{n}{r}x^{2n-3r}.$$

Let $x^6$ come from index $r$, so $2n - 3r = 6$. Then $x^3$ comes from index $r+1$. The ratio of coefficients is

$$\frac{\binom{n}{r}}{\binom{n}{r+1}} = \frac{r+1}{n-r} = \frac{3}{2} \Rightarrow 5r + 2 = 3n.$$

With $3r = 2n - 6$: $5 \cdot \frac{2n-6}{3} + 2 = 3n \Rightarrow 10n - 24 = 9n \Rightarrow n = 24$, $r = 14$. Both $r = 14$ and $r + 1 = 15$ lie in $0 \le r \le n$, so both terms occur.
Inverting the ratio, $\frac{n-r}{r+1} = \frac{3}{2}$, gives $n = \frac{21}{4}$, which is not an integer. The ratio must be taken in the stated order.
**Answer: (D)**

**Common mistake:** Inverting the coefficient ratio $\binom{n}{r}/\binom{n}{r+1}$, or writing the general term's exponent as $2n - r$ instead of $2n - 3r$.

**Q.56** Sequences – geometric progression • Foundation • 80 s **Answer: B**

A positive geometric progression has second term 6 and fourth term 24. Find its fifth term.

(A) 12 (B) 48 (C) 96 (D) 36

**Solution**

$$\frac{T_4}{T_2} = r^2 = \frac{24}{6} = 4.$$

Every term is positive, so $r = 2$ (reject $r = -2$). Then $T_5 = T_4\, r = 24 \times 2 = 48$.
Taking $r = 4$ from $T_4/T_2$, without noticing that the gap is two steps, gives 96 (C).
**Answer: (B)**

**Common mistake:** Treating $T_4/T_2$ as $r$ instead of $r^2$, which gives $r = 4$ and $T_5 = 96$.

**Q.57** Functions – domain, range and graphs • Standard • 150 s **Answer: D**

Find the number of integers $x$ in the domain of $\dfrac{\sqrt{(x-1)(5-x)}}{\ln(x-2)}$.

(A) 3 (B) 4 (C) 1 (D) 2

**Solution**

Radical: $(x-1)(5-x) \geq 0 \Rightarrow 1 \leq x \leq 5$.

Logarithm: $x-2>0 \Rightarrow x>2$. Denominator: $\ln(x-2) \neq 0 \Rightarrow x \neq 3$.

Domain: $(2,5] \setminus \{3\}$. The integers in it are $4$ and $5$, so there are 2 of them.

Forgetting that $\ln(x-2)$ vanishes at $x=3$ gives 3 integers (A).

**Answer: (D)**

**Common mistake:** Forgetting that the denominator $\ln(x-2)$ is zero at $x=3$, which counts $3, 4, 5$.

**Q.58** Integration – integration by parts • Foundation • 80 s **Answer: C**

Evaluate $\displaystyle\int_0^1 x\,e^x\,dx$.

(A) 2 (B) 0 (C) 1 (D) 3

**Solution**

Integrate by parts with $u=x$ and $dv=e^x\,dx$:

$$\int xe^x\,dx = xe^x - \int e^x\,dx = (x-1)e^x.$$

$$\int_0^1 xe^x\,dx = \left[(x-1)e^x\right]_0^1 = 0-(-1) = 1.$$

Dropping the lower-limit term $(0-1)e^0 = -1$ gives 0 (B).

**Answer: (C)**

**Common mistake:** Evaluating only the upper limit, or taking $(x-1)e^x$ to be 0 at $x=0$, which gives 0.

**Q.59** Differential equations – order and degree • Standard • 150 s **Answer: A**

For $\left(\dfrac{d^2y}{dx^2}\right)^3 + \sqrt{1+\left(\dfrac{dy}{dx}\right)^2} = 0$, eliminating the radical produces a polynomial differential equation. What is its degree?

(A) 6 (B) 8 (C) 5 (D) 7

**Solution**

Isolate the radical and square:

$$(y'')^3 = -\sqrt{1+(y')^2} \;\Rightarrow\; (y'')^6 = 1+(y')^2.$$

The highest-order derivative is $y''$ and its highest power is 6, so the degree is 6. (Squaring may add solutions, but the question asks only for the degree of the polynomial form.)

The value 3 is the power of $y''$ before the radical is removed. It is not the degree, because the equation is not yet a polynomial in the derivatives.

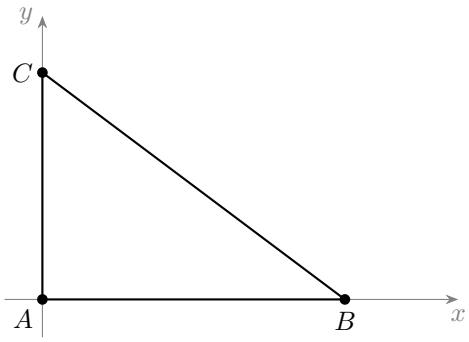
**Answer: (A)**

**Common mistake:** Reporting degree 3 without first removing the radical.

## Q.60

Coordinate geometry – distance and section formula • Challenging • 220 s **Answer: B**

$A = (0,0)$, $B = (8,0)$, $C = (0,6)$. $P$ on $BC$ satisfies $BP : PC = 1 : 3$. The perpendicular bisector of $AP$ meets the $x$-axis at $T$. Find the ratio of the areas of triangles $PTC$ and $ABC$.




(A) $\frac{231}{256}$ (B) $\frac{231}{512}$

(C) $\frac{77}{256}$ (D) $\frac{17}{128}$

**Solution**

Section formula ($BP : PC = 1 : 3$):

$$P = \frac{3B + 1 \cdot C}{4} = \left(6, \tfrac{3}{2}\right).$$

$T = (t, 0)$ is on the perpendicular bisector of $AP$, so $TA = TP$:

$$t^2 = (t-6)^2 + \tfrac{9}{4} \;\Rightarrow\; 12t = \tfrac{153}{4} \;\Rightarrow\; t = \tfrac{51}{16}.$$

Twice the area of $PTC$ is given by the determinant:

$$\left|6(0-6) + \tfrac{51}{16}\left(6 - \tfrac{3}{2}\right) + 0\right| = \left|-36 + \tfrac{459}{32}\right| = \tfrac{693}{32}.$$

So $[PTC] = \frac{693}{64}$ and $[ABC] = \frac{1}{2} \cdot 8 \cdot 6 = 24$:

$$\frac{[PTC]}{[ABC]} = \frac{693}{64 \cdot 24} = \frac{231}{512}.$$

Forgetting to halve the determinant gives $231/256$ (A).

**Answer: (B)**

**Common mistake:** Using the determinant value $693/32$ as the area without halving (giving $231/256$), or placing $P$ with the ratio reversed, $P = (2, \frac{9}{2})$.

## Q.61

3D geometry – distance and section in space • Foundation • 80 s **Answer: C**

$A = (1, -2, 3)$ and $B = (5, 4, -1)$. Find the square of the distance from the origin to their midpoint.

(A) 12 (B) 10 (C) 11 (D) 13

**Solution**

Midpoint: $M = \left(\frac{1+5}{2}, \frac{-2+4}{2}, \frac{3-1}{2}\right) = (3, 1, 1)$.

$$OM^2 = 3^2 + 1^2 + 1^2 = 11.$$

Averaging the squared distances, $\frac{1}{2}(OA^2 + OB^2) = \frac{1}{2}(14 + 42) = 28$, is a different quantity and is not $OM^2$.

**Answer: (C)**

**Common mistake:** Forgetting to halve the coordinate sum, or squaring before averaging.

**Q.62** Vectors – components and addition • Standard • 150 s **Answer: C**

Nonparallel vectors $\vec{a}, \vec{b}$ determine $\vec{u} = 2\vec{a} + \vec{b}$ and $\vec{v} = \vec{a} - \vec{b}$. A point $P$ has position vector $\lambda\vec{u} + (1-\lambda)\vec{v}$. If its $\vec{a}$- and $\vec{b}$-coefficients are equal, find $\lambda$.

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$

(C) 2 (D) $-2$

**Solution**

$$\lambda(2\vec{a} + \vec{b}) + (1-\lambda)(\vec{a} - \vec{b}) = (1+\lambda)\vec{a} + (2\lambda - 1)\vec{b}.$$

Since $\vec{a}, \vec{b}$ are nonparallel, the coefficients are unique. Setting them equal:

$$1 + \lambda = 2\lambda - 1 \;\Rightarrow\; \lambda = 2.$$

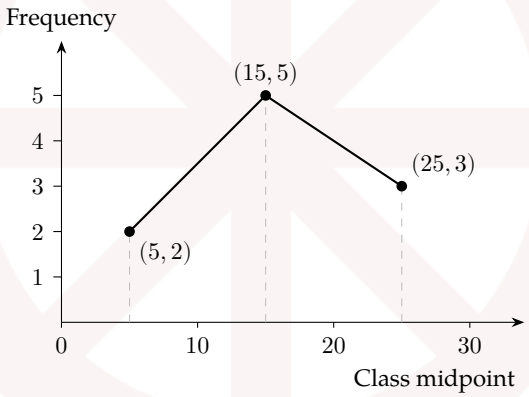
($P$ lies beyond the segment $UV$, since $\lambda > 1$.) Expanding $(1-\lambda)(-\vec{b})$ as $-\vec{b} - \lambda\vec{b}$ is a sign slip. It gives the $\vec{b}$-coefficient $-1$ and leads to $\lambda = -2$ (D).

**Answer: (C)**

**Common mistake:** Expanding $(1-\lambda)(-\vec{b})$ with the wrong sign on the $\lambda\vec{b}$ term, which gives $\lambda = -2$.

**Q.63** Statistics – mean of grouped data • Foundation • 150 s **Answer: D**

Classes $[0, 10)$, $[10, 20)$, $[20, 30)$ have frequencies $2, 5, 3$. Using class midpoints, find the mean.




(A) 18 (B) 15 (C) 17 (D) 16

**Solution**

The class midpoints are $5, 15, 25$ and the total frequency is $N = 10$.

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{2(5) + 5(15) + 3(25)}{10} = \frac{160}{10} = 16.$$

15 (B) is only the midpoint of the modal class. Lower limits would give 11, and upper limits would give 21.

**Answer: (D)**

**Common mistake:** Using class lower limits instead of midpoints.

**Q.64** Trigonometry – identity transformations • Standard • 150 s **Answer: C**

For an acute $\theta$, $\sin\theta + \cos\theta = \sqrt{\frac{3}{2}}$. Find $(\tan\theta - \cot\theta)^2$.

(A) 4 (B) 14 (C) 12 (D) 16

**Solution**

Squaring: $1 + 2\sin\theta\cos\theta = \frac{3}{2} \Rightarrow \sin\theta\cos\theta = \frac{1}{4}$.

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = 4, \qquad \tan\theta\cot\theta = 1.$$

$$(\tan\theta - \cot\theta)^2 = (\tan\theta + \cot\theta)^2 - 4\tan\theta\cot\theta = 16 - 4 = 12.$$

16 (D) is $(\tan\theta + \cot\theta)^2$. It comes from forgetting the $-4\tan\theta\cot\theta$ term.

**Answer: (C)**

**Common mistake:** Reporting $(\tan\theta + \cot\theta)^2 = 16$ and forgetting to subtract $4\tan\theta\cot\theta$.

## Q.65

Sets – power sets • Challenging • 220 s **Answer: B**

Sets $A$ and $B$ satisfy $|A| = |B| = 5$ and $|A \cap B| = 2$. How many ordered pairs $(X, Y)$ satisfy $X \subseteq A$, $Y \subseteq B$, $X \cap Y = \emptyset$ and $|X \cup Y| = 3$?

(A) 110 (B) 104 (C) 160 (D) 80

### Solution

$A \setminus B$ has 3 elements (each can only be omitted or put in $X$). $B \setminus A$ has 3 elements (each omitted or in $Y$). Each of the 2 common elements has three states: omitted, in $X$, or in $Y$. It cannot be in both, because $X \cap Y = \emptyset$.

Marking each chosen element with $t$, the generating function is $(1+t)^6(1+2t)^2 = (1+t)^6(1+4t+4t^2)$. We need the coefficient of $t^3$:

$$\binom{6}{3} + 4\binom{6}{2} + 4\binom{6}{1} = 20 + 60 + 24 = 104.$$

Treating each common element as a single yes/no choice gives only $\binom{8}{3} = 56$. The factor $(1+2t)$ records that a chosen common element must also be assigned to $X$ or to $Y$.

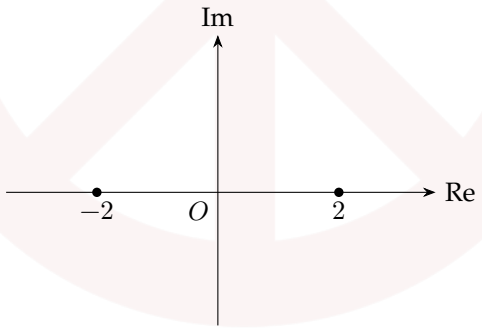
**Answer: (B)**

**Common mistake:** Treating each common element as a single choice (forgetting it can go to $X$ or to $Y$), or allowing it in both sets despite $X \cap Y = \emptyset$.

## Q.66

Complex numbers – Argand-plane locus • Foundation • 80 s **Answer: A**

If $z = x + iy$ satisfies $|z - 2| = |z + 2|$, which is its locus?




(A) $x = 0$ (B) $y = 0$ (C) $x^2 + y^2 = 4$ (D) $x = 2$

### Solution

$|z - 2| = |z + 2|$ means that $z$ is equidistant from 2 and $-2$.

$$(x-2)^2 + y^2 = (x+2)^2 + y^2 \;\Rightarrow\; -4x = 4x \;\Rightarrow\; x = 0.$$

The locus is the perpendicular bisector of the segment joining $-2$ and 2, which is the imaginary axis.

$y = 0$ (B) is the line through the two points. It is not their perpendicular bisector.

**Answer: (A)**

**Common mistake:** Taking the line through the two points ($y = 0$) instead of their perpendicular bisector.

**Q.67** Matrices – matrix equations • Challenging • 150 s **Answer: D**

Let $A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$. How many real $2 \times 2$ matrices $X$ satisfy $AX + XA = \begin{pmatrix} 4 & 0 \\ 0 & -4 \end{pmatrix}$, $\mathrm{tr}(X) = 0$ and $\det(X) = -1$?

(A) 2 (B) Infinitely many
(C) 1 (D) 0

**Solution**

Let $X = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$. Then

$$AX = \begin{pmatrix} p+r & q+s \\ -r & -s \end{pmatrix}, \quad XA = \begin{pmatrix} p & p-q \\ r & r-s \end{pmatrix}, \quad AX + XA = \begin{pmatrix} 2p+r & p+s \\ 0 & r-2s \end{pmatrix}.$$

We need $p+s=0$, $2p+r=4$ and $r-2s=-4$. With $s=-p$, the last equation becomes $2p+r=-4$. This contradicts $2p+r=4$.

So no matrix $X$ satisfies the equation, whatever the trace and determinant conditions. The count is $0$.

If you write $XA$ as $AX$ (assuming commutativity), you get $2AX = \begin{pmatrix} 4 & 0 \\ 0 & -4 \end{pmatrix}$, which looks solvable. That slip makes the system look solvable and hides the inconsistency.

**Answer: (D)**

**Common mistake:** Treating $AX + XA$ as $2AX$ (assuming $A$ and $X$ commute), which hides the inconsistency.

**Q.68** Combinations – selection with restrictions • Standard • 150 s **Answer: B**

From 5 men and 4 women a committee of 4 is chosen. Two specified women refuse to serve together. How many committees contain exactly two women?

(A) 40 (B) 50 (C) 60 (D) 100

**Solution**

Men: $\binom{5}{2} = 10$ ways. Women: $\binom{4}{2} = 6$ pairs, of which exactly one (the two specified women) is forbidden, leaving 5.

$$10 \times 5 = 50.$$

Ignoring the restriction gives $10 \times 6 = 60$ (C).

**Answer: (B)**

**Common mistake:** Ignoring the restriction (giving 60), or subtracting the forbidden pair from the total of all committees instead of only from the two-woman committees.

**Q.69** Binomial theorem – middle term • Standard • 150 s **Answer: B**

For positive $a, b$, the two middle terms of $(a+b)^7$ are equal and their sum is 8960. Find $a+b$.

(A) 8 (B) 4 (C) 16 (D) 2

**Solution**

The middle terms are $T_4 = \binom{7}{3}a^4b^3 = 35a^4b^3$ and $T_5 = \binom{7}{4}a^3b^4 = 35a^3b^4$.

If they are equal, then $35a^3b^3(a-b) = 0$, so $a = b$ (since $a, b > 0$). Their sum is $70a^7 = 8960$, so $a^7 = 128$ and $a = b = 2$.

$$a + b = 4.$$

2 (D) is the value of $a$ alone, not $a+b$.

**Answer: (B)**

**Common mistake:** Stopping at $a = 2$ and reporting it instead of $a+b$, or taking the middle term of $(a+b)^7$ as a single term.

**Q.70** Sequences – insertion of means • Standard • 220 s **Answer: A**

Between positive numbers $a < b$, two geometric means $p, q$ and one arithmetic mean $A$ are inserted. If $p + q = 18$ and $A = 21$, find $b - a$.

(A) 39 (B) 36 (C) 27 (D) 42

**Solution**

With common ratio $r > 1$: $p = ar$, $q = ar^2$, $b = ar^3$. From $A = 21$, $a + b = 42$. So

$$\frac{a(1+r^3)}{a(r+r^2)} = \frac{42}{18} = \frac{7}{3} \Rightarrow \frac{r^2 - r + 1}{r} = \frac{7}{3} \Rightarrow 3r^2 - 10r + 3 = 0.$$

So $r = 3$ ($r = \frac{1}{3}$ is rejected since $a < b$). Then $a = \dfrac{18}{3+9} = \dfrac{3}{2}$ and $b = \dfrac{81}{2}$, giving $b - a = 39$.

Taking $b = ar^2$ gives the wrong ratio. Two inserted geometric means create three ratios between $a$ and $b$, so $b = ar^3$.

**Answer: (A)**

**Common mistake:** Taking $b = ar^2$ for two inserted geometric means (there are three ratios, so $b = ar^3$).

**Q.71** Limits – standard algebraic limits • Foundation • 150 s **Answer: -2**

Find $\lim\limits_{x \to 0} \dfrac{\sqrt{1+4x} - 1 - 2x}{x^2}$. (The answer may be negative.)

**Solution**

Rationalise: $\sqrt{1+4x} - 1 = \dfrac{4x}{\sqrt{1+4x}+1}$. Then

$$\sqrt{1+4x} - 1 - 2x = 2x \cdot \frac{2 - (\sqrt{1+4x}+1)}{\sqrt{1+4x}+1} = 2x \cdot \frac{1 - \sqrt{1+4x}}{\sqrt{1+4x}+1} = \frac{-8x^2}{\left(\sqrt{1+4x}+1\right)^2}.$$

$$\lim_{x \to 0} \frac{-8}{\left(\sqrt{1+4x}+1\right)^2} = \frac{-8}{4} = -2.$$

Check with the binomial series: $\sqrt{1+4x} = 1 + 2x - 2x^2 + \cdots$, so the $x^2$ coefficient of the numerator is $-2$.

**Answer:** $-2$

**Common mistake:** Dropping the minus sign of the $-\frac{1}{8}u^2$ term in $\sqrt{1+u} = 1 + \frac{u}{2} - \frac{u^2}{8} + \cdots$ and answering 2.

**Q.72** Integration – partial fractions • Standard • 150 s **Answer: 17**

Let $I = \displaystyle\int_0^1 \frac{x}{(x+1)(x+2)}\,dx$. If $I = \ln \dfrac{a}{b}$ with positive coprime integers $a, b$, find $a + b$.

**Solution**

$$\frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}.$$

$$I = \Big[ -\ln(x+1) + 2\ln(x+2) \Big]_0^1 = -\ln 2 + 2\ln\frac{3}{2} = \ln\frac{9}{8}.$$

$a = 9$, $b = 8$ (coprime), so $a + b = 17$.

**Answer:** 17

**Common mistake:** Getting the partial-fraction signs wrong (for example $\frac{1}{x+1} - \frac{2}{x+2}$), which gives $\ln\frac{8}{9}$, or forgetting to subtract the value at $x = 0$.

**Q.73** Differential equations – separable equations • Standard • 150 s **Answer: 4**

A positive solution satisfies $y' = \dfrac{x}{y}$ and $y(0) = 2$. Let $a > 0$ satisfy $y(a) = 2a$. Find $3a^2$.

**Solution**

Separate: $y\,dy = x\,dx \Rightarrow y^2 = x^2 + C$. From $y(0) = 2$, $C = 4$, so $y = \sqrt{x^2+4}$ (positive branch).

$$y(a) = 2a \;\Rightarrow\; 4a^2 = a^2 + 4 \;\Rightarrow\; 3a^2 = 4.$$

Since $a > 0$ and $y > 0$, the unsquared condition $\sqrt{a^2+4} = 2a$ holds.

**Answer:** 4

**Common mistake:** Dropping the factor $\frac{1}{2}$ on only one side when integrating ($y^2 = \frac{x^2}{2} + C$), or fixing the constant before integrating.

## Q.74

Coordinate geometry – locus from a geometric constraint • Standard • 150 s **Answer: 64**

$P = (x, y)$ satisfies $PA^2 + PB^2 = 50$ for $A = (-3, 0)$, $B = (3, 0)$. A line through the origin with slope $1$ meets this locus at $U, V$. Find $|UV|^2$.

**Solution**

$$(x+3)^2 + y^2 + (x-3)^2 + y^2 = 50 \;\Rightarrow\; 2x^2 + 2y^2 + 18 = 50 \;\Rightarrow\; x^2 + y^2 = 16.$$

The locus is a circle with centre $O$ and radius $4$. The line $y = x$ passes through the centre, so $UV$ is a diameter: $|UV| = 8$ and $|UV|^2 = 64$.

**Answer:** 64

**Common mistake:** Forgetting the $2 \cdot 9 = 18$ constant when expanding (giving $x^2 + y^2 = 25$), or reporting $|UV|$ or the radius instead of $|UV|^2$.

## Q.75

3D geometry – direction cosines and ratios • Challenging • 220 s **Answer: 26**

A direction vector is $(1, t, t^2)$. Its angle with the positive $y$-axis has cosine $\frac{1}{\sqrt{3}}$. Let $\hat{u}$ be its unit vector with positive first component, and $\hat{v}$ the unit vector along $(1, -2, 2)$. Find $27\,|\hat{u} \times \hat{v}|^2$.

**Solution**

$$\frac{t}{\sqrt{1 + t^2 + t^4}} = \frac{1}{\sqrt{3}} \;\Rightarrow\; t > 0,\; 3t^2 = 1 + t^2 + t^4 \;\Rightarrow\; (t^2 - 1)^2 = 0 \;\Rightarrow\; t = 1.$$

So $\hat{u} = \dfrac{(1,1,1)}{\sqrt{3}}$ and $\hat{v} = \dfrac{(1,-2,2)}{3}$, with $\hat{u} \cdot \hat{v} = \dfrac{1 - 2 + 2}{3\sqrt{3}} = \dfrac{1}{3\sqrt{3}}$.

$$|\hat{u} \times \hat{v}|^2 = 1 - (\hat{u} \cdot \hat{v})^2 = 1 - \frac{1}{27} = \frac{26}{27}, \qquad 27|\hat{u} \times \hat{v}|^2 = 26.$$

**Answer:** 26

**Common mistake:** Using the unnormalised vectors $(1, 1, 1)$ and $(1, -2, 2)$ in the cross product, or taking $t = -1$ (which gives a negative cosine with the $y$-axis).