Physics

HSC Board Exam February–March 2027 – Practice Paper

Model Answer Paper

Time: 3 hours **Max. Marks: 70**

Model answers show one correct method; any other correct method gets full marks.

SECTION A – Q.1 Select and write the correct answer for the following multiple choice type of questions: **[10]**

(i) A uniform disc rolls without slipping on a horizontal surface. The fraction of its total kinetic energy that is rotational is _____.

(A) 1/2 (B) 1/3
(C) 2/3 (D) 1/4

Answer (1 mark)
(B) 1/3
For a disc $k^2/r^2 = 1/2$. $KE_{rot}/KE_{total} = (k^2/r^2)/(1 + k^2/r^2) = (1/2)/(3/2) = 1/3$.

(ii) In a cyclotron, the frequency of revolution of a charged particle in the uniform magnetic field does NOT depend on its _____.

(A) charge (B) mass
(C) magnetic induction (D) speed

Answer (1 mark)
(D) speed
$f = qB/(2\pi m)$ contains charge, mass and field but not the speed; a faster particle simply moves on a larger circle.

(iii) In linear S.H.M., the phase difference between the displacement and the velocity of the particle is _____.

(A) 0 (B) π/4 rad
(C) π/2 rad (D) π rad

Answer (1 mark)
(C) π/2 rad
$x = A \sin(\omega t + \alpha)$, $v = A\omega \cos(\omega t + \alpha) = A\omega \sin(\omega t + \alpha + \pi/2)$, so velocity leads displacement by π/2.

(iv) A Young's double-slit set-up gives a fringe width W in air. If the entire set-up is immersed in water of refractive index 4/3, the fringe width becomes _____.

(A) 3W/4 (B) 4W/3
(C) W (D) 9W/16

Answer (1 mark)
(A) 3W/4
$W = \lambda D/d$ and λ in water = $\lambda/n = 3\lambda/4$, so the new fringe width is 3W/4.

(v) The sensitivity of a potentiometer can be increased by _____.

(A) increasing the current through the wire
(B) decreasing the length of the wire
(C) increasing the length of the potentiometer wire
(D) increasing the potential gradient

Answer (1 mark)
(C) increasing the length of the potentiometer wire
A longer wire (same driving emf) has a smaller potential gradient, so a small p.d. corresponds to a larger balancing length.

(vi) The magnetic susceptibility of a diamagnetic substance is _____.

(A) small and positive
(B) small and negative
(C) large and positive
(D) zero

Answer (1 mark)
(B) small and negative
A diamagnetic substance is weakly magnetised opposite to the applied field, so χ is small (about -10^{-5}) and negative.

(vii) The average power consumed over one complete cycle in a purely inductive a.c. circuit is _____.

(A) $e_{rms}\, i_{rms}$
(B) $e_0\, i_0$
(C) $e_{rms}\, i_{rms}/\sqrt{2}$
(D) zero

Answer (1 mark)
(D) zero
Current lags the emf by $\pi/2$, so $P = e_{rms}\, i_{rms} \cos(\pi/2) = 0$ (wattless current).

(viii) The accelerating potential difference for an electron is made four times. Its de Broglie wavelength becomes _____.

(A) half
(B) double
(C) one-fourth
(D) four times

Answer (1 mark)
(A) half
$\lambda = h/\sqrt{(2meV)} \propto 1/\sqrt{V}$; $V \to 4V$ gives $\lambda \to \lambda/\sqrt{4} = \lambda/2$.

(ix) A full-wave rectifier is supplied with a.c. of frequency 50 Hz. The frequency of the ripple in its output is _____.

(A) 25 Hz
(B) 50 Hz
(C) 100 Hz
(D) 200 Hz

Answer (1 mark)
(C) 100 Hz
Both half cycles give an output pulse, so there are two pulses per input cycle: $2 \times 50 = 100$ Hz.

(x) According to Bohr's model, the ratio of the radius of the third orbit to that of the first orbit of the hydrogen atom is _____.

(A) 3 : 1
(B) 9 : 1
(C) 1 : 9
(D) 27 : 1

Answer (1 mark)
(B) 9 : 1
$r_n \propto n^2$, so $r_3/r_1 = 3^2/1^2 = 9$.

Q.2 Answer the following questions: **[8]**

(i) Write the dimensions of the coefficient of viscosity.

> **Answer** (1 mark)
> $[M^1L^{-1}T^{-1}]$
> $\eta = F/(A \cdot dv/dx) \rightarrow [MLT^{-2}]/([L^2][T^{-1}]) = [ML^{-1}T^{-1}]$.

(ii) What is the change in internal energy of an ideal gas in an isothermal process?

> **Answer** (1 mark)
> Zero.
> Internal energy of an ideal gas depends only on temperature; in an isothermal process T is constant, so $\Delta U = 0$ (and $Q = W$).

(iii) What is the distance between a node and the next antinode in a stationary wave of wavelength λ?

> **Answer** (1 mark)
> $\lambda/4$.
> Adjacent nodes (or antinodes) are $\lambda/2$ apart; a node and the next antinode are $\lambda/4$ apart.

(iv) Write the SI unit of electric polarization.

> **Answer** (1 mark)
> C/m^2 (coulomb per square metre).
> Polarization = dipole moment per unit volume = $C\ m/m^3 = C\ m^{-2}$.

(v) Write the expression for the magnetic induction inside a long solenoid having n turns per unit length and carrying current I.

> **Answer** (1 mark)
> $B = \mu_0 nI$
> (Uniform field along the axis, well inside the solenoid; $n = N/l$.)

(vi) Lenz's law is a consequence of the law of conservation of which physical quantity?

> **Answer** (1 mark)
> Energy.
> The induced current opposes the change producing it, so mechanical work must be done to keep inducing it; this work appears as electrical energy.

(vii) What is the output of a two-input NAND gate when both inputs are 1?

> **Answer** (1 mark)
> 0.
> $Y = (A \cdot B)' = (1 \cdot 1)' = 1' = 0$.

(viii) State the energy equivalent of 1 atomic mass unit (1 u) in MeV.

> **Answer** (1 mark)
> 931.5 MeV (≈ 931 MeV).
> From $E = mc^2$ with $1\ u = 1.66 \times 10^{-27}$ kg.

SECTION B Attempt any EIGHT questions of the following: (2 marks each) **[16]**

Q.3 The angular speed of a flywheel increases uniformly from 120 r.p.m. to 300 r.p.m. in 6 s. Find (a) its angular acceleration and (b) the number of revolutions it makes in this time.

Answer (2 marks)

Data: $n_1 = 120$ r.p.m. $= 2$ r.p.s., $n_2 = 300$ r.p.m. $= 5$ r.p.s., $t = 6$ s

$\omega_1 = 2\pi n_1 = 4\pi$ rad/s, $\omega_2 = 2\pi n_2 = 10\pi$ rad/s

(a) $\alpha = (\omega_2 - \omega_1)/t = (10\pi - 4\pi)/6 = \pi$ rad/s^2 = **3.14 rad/s^2**

(b) $\theta = ((\omega_1 + \omega_2)/2)\, t = (14\pi/2) \times 6 = 42\pi$ rad

Number of revolutions $= \theta/2\pi = 42\pi/2\pi =$ **21**

Marking: Conversion to rad/s and $\alpha = \pi$ rad/s^2 – 1 mark · $\theta = 42\pi$ rad and 21 revolutions – 1 mark

Examiner tip: Convert r.p.m. to rad/s first ($\omega = 2\pi n$ with n in r.p.s.). Many students leave α in r.p.m. per second.

Q.4 Define surface tension and surface energy. Show that the surface tension of a liquid is numerically equal to its surface energy per unit area.

Answer (2 marks)

Surface tension (T): the tangential force per unit length acting on either side of an imaginary line drawn on the free surface of a liquid, $T = F/l$.

Surface energy: the extra potential energy of the molecules of the surface layer (compared with the interior), per unit area.

Proof: Take a rectangular wire frame with a sliding wire of length l, holding a liquid film. The film has two surfaces, so the force on the wire is $F = 2Tl$.

If the wire is pulled out slowly (isothermally) through dx, work done $W = F\,dx = 2Tl\,dx$.

Increase in surface area $\Delta A = 2l\,dx$ (both surfaces).

Surface energy per unit area $= W/\Delta A = 2Tl\,dx/(2l\,dx) = T$.

Hence surface tension = surface energy per unit area.

Marking: Both definitions – ½ mark · $W = 2Tl\,dx$ and $\Delta A = 2l\,dx$ (two surfaces) – 1 mark · Conclusion $W/\Delta A = T$ – ½ mark

Examiner tip: Remember the film has TWO free surfaces; missing the factor 2 in force or area is the usual slip.

Q.5 What is meant by the resolving power of a telescope? State two ways of increasing it.

Answer (2 marks)

Resolving power of a telescope is its ability to show two closely placed distant objects (e.g. stars) as separate. It is the reciprocal of the limiting angle of resolution:

R.P. $= 1/\theta = D/(1.22\,\lambda)$, where D = diameter (aperture) of the objective and λ = wavelength of light.

Ways to increase it:

1. Use an objective of larger aperture (diameter D).
2. Use light of shorter wavelength λ.

Marking: Definition with R.P. $= D/1.22\lambda$ – 1 mark · Two correct ways – ½ mark each

Examiner tip: Increasing the focal length or magnifying power does NOT change resolving power; examiners mark that wrong.

Q.6 Using the first law of thermodynamics, explain why a gas cools when it expands adiabatically.

Answer (2 marks)

First law: $Q = \Delta U + W$.

In an adiabatic process no heat is exchanged with the surroundings, so $Q = 0$ and $\Delta U = -W$.

During expansion the gas does work on the surroundings, so W is positive and ΔU is negative.

The internal energy of an ideal gas depends only on temperature, so a decrease in U means a

fall in temperature. Hence the gas cools.
(Example: air escaping from a burst tyre feels cool.)

Marking: Q = 0 ⇒ ΔU = –W – 1 mark · W > 0 in expansion ⇒ ΔU < 0 ⇒ temperature falls – 1 mark

Examiner tip: State the sign convention (W positive when work is done BY the gas).

Q.7 A particle performs linear S.H.M. of amplitude 4 cm and period π/5 s. Find its maximum velocity and maximum acceleration.

Answer (2 marks)

Data: A = 4 cm = 0.04 m, T = π/5 s

$\omega = 2\pi/T = 2\pi/(\pi/5) = 10$ rad/s

$v_{max} = A\omega = 0.04 \times 10 =$ **0.4 m/s**

$a_{max} = A\omega^2 = 0.04 \times 100 =$ **4 m/s^2**

Marking: ω = 10 rad/s – ½ mark · v_{max} = 0.4 m/s – ½ mark · a_{max} = 4 m/s^2 – 1 mark

Examiner tip: Write units in SI; 'v = 40' without units loses the mark.

Q.8 A pipe 17 cm long is closed at one end. Neglecting end correction, find the frequency of its fundamental mode and of its first overtone. (Speed of sound in air = 340 m/s)

Answer (2 marks)

For a pipe closed at one end, L = λ/4 in the fundamental mode.

$n = v/(4L) = 340/(4 \times 0.17) = 340/0.68 =$ **500 Hz**

A closed pipe gives only odd harmonics, so the first overtone is the third harmonic:

$n_1 = 3n = 3 \times 500 =$ **1500 Hz**

Marking: n = v/4L = 500 Hz – 1 mark · First overtone = 3n = 1500 Hz – 1 mark

Examiner tip: First overtone of a closed pipe is 3n, not 2n (2n belongs to an open pipe).

Q.9 A parallel plate air capacitor has plates of area 0.02 m^2 separated by 1.77 mm. Find its capacitance and the charge on each plate when it is connected to a 50 V supply. ($\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N m^2)

Answer (2 marks)

$C = \varepsilon_0 A/d = (8.85 \times 10^{-12} \times 0.02)/(1.77 \times 10^{-3})$

$= (8.85/1.77) \times 0.02 \times 10^{-9} = 5 \times 0.02 \times 10^{-9}$

$C =$ **1×10^{-10} F = 100 pF**

$Q = CV = 1 \times 10^{-10} \times 50 =$ **5×10^{-9} C**

Marking: Formula and C = 10^{-10} F – 1 mark · Q = 5×10^{-9} C – 1 mark

Examiner tip: Convert 1.77 mm to metres; notice 8.85/1.77 = 5 to avoid long working.

Q.10 State Kirchhoff's current law and voltage law, mentioning the sign conventions used.

Answer (2 marks)

1. Kirchhoff's current law (junction law): The algebraic sum of the currents at any junction of a circuit is zero, ΣI = 0. Currents entering the junction are taken positive and currents leaving it negative. (It follows from conservation of charge.)
2. Kirchhoff's voltage law (loop law): In any closed loop of a circuit, the algebraic sum of the products of current and resistance (IR) and the emfs is zero, ΣIR + ΣE = 0. Moving around the loop in the chosen direction, IR is taken negative when we move along the current and positive against it; an emf is positive when we go from – to + terminal inside the cell. (It follows from conservation of energy.)

Marking: Current law with sign convention – 1 mark · Voltage law with sign convention – 1 mark

Examiner tip: Any consistent sign convention is accepted, but it must be stated.

Q.11 A circular coil of 50 turns and radius 6.28 cm carries a current of 2 A. Find the magnetic induction at its centre. ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A m, $\pi = 3.14$)

Answer (2 marks)

$B = \mu_0 NI/(2R)$

$= (4 \times 3.14 \times 10^{-7} \times 50 \times 2)/(2 \times 6.28 \times 10^{-2})$

$= (12.56 \times 10^{-5})/(12.56 \times 10^{-2})$

$B = \mathbf{1 \times 10^{-3}\ T}$ (= 1 mT), along the axis of the coil.

Marking: Correct formula $B = \mu_0 NI/2R$ – ½ mark · Substitution – ½ mark · Answer with unit – 1 mark

Examiner tip: Do not forget N; and R must be in metres.

Q.12 An induction cooktop quickly heats a steel vessel kept on it, but a glass bowl kept on it stays cool. Explain, naming the currents responsible.

Answer (2 marks)

1. The coil below the cooktop carries a high-frequency alternating current, which produces a rapidly changing magnetic field.
2. This changing flux induces closed loops of current, called **eddy currents**, in the body of any conductor placed in it (Faraday's law).
3. The steel base is a conductor, so large eddy currents flow in it and produce heat (I^2R) – the vessel heats up.
4. Glass is an insulator, so no eddy currents are induced in it and it does not get heated by the cooktop.

Marking: Changing flux induces eddy currents – 1 mark · Conductor heats (I^2R), insulator glass has no eddy currents – 1 mark

Examiner tip: Name the effect 'eddy currents' and say why glass is different (insulator).

Q.13 Light of wavelength 4140 Å falls on a metal surface of work function 1.8 eV. Find the maximum kinetic energy of the photoelectrons (in eV) and the stopping potential. ($h = 6.63 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m/s, 1 eV = 1.6×10^{-19} J)

Answer (2 marks)

Energy of a photon $E = hc/\lambda = (6.63 \times 10^{-34} \times 3 \times 10^8)/(4140 \times 10^{-10})$

$= 4.8 \times 10^{-19}$ J $= 4.8 \times 10^{-19}/1.6 \times 10^{-19}$ eV $= 3.0$ eV

$K.E._{max} = h\nu - \phi_0 = 3.0 - 1.8 = \mathbf{1.2\ eV}$

$eV_0 = K.E._{max} \Rightarrow$ stopping potential $V_0 = \mathbf{1.2\ V}$

Marking: Photon energy 3.0 eV – 1 mark · $K.E._{max} = 1.2$ eV and $V_0 = 1.2$ V – 1 mark

Examiner tip: Convert joule to eV before subtracting the work function; stopping potential is in volt, not eV.

Q.14 State Curie's law for a paramagnetic material. A paramagnetic salt kept in a fixed external magnetic field has magnetization 3.6×10^3 A/m at –73 °C. Find its magnetization in the same field at 27 °C.

Answer (2 marks)

Curie's law: The magnetization of a paramagnetic material is directly proportional to the external magnetic field and inversely proportional to its absolute temperature:

$M_z = C\, B_{ext}/T$, where C is the Curie constant. (Equivalently $\chi \propto 1/T$.)

(Reason: a higher temperature means greater thermal agitation, so fewer atomic dipoles stay aligned with the field.)

Numerical: $T_1 = -73 + 273 = 200$ K, $T_2 = 27 + 273 = 300$ K, B_{ext} same $\Rightarrow M \propto 1/T$

$M_2 = M_1 \times T_1/T_2 = 3.6 \times 10^3 \times 200/300 =$ **2.4×10^3 A/m**

Marking: Statement of Curie's law ($M_z = C\,B_{ext}/T$) – 1 mark · Temperatures in K and $M_2 = 2.4 \times 10^3$ A/m – 1 mark

Examiner tip: Convert °C to kelvin before taking the ratio; magnetization falls as temperature rises.

SECTION C Attempt any EIGHT questions of the following: (3 marks each) **[24]**

Q.15 Obtain an expression for the total kinetic energy of a body rolling without slipping on a horizontal surface, in terms of its linear speed and radius of gyration.

Answer (3 marks)

A rolling body has translational motion of its centre of mass and rotational motion about an axis through the centre of mass.

Let m = mass, r = radius, k = radius of gyration, v = speed of centre of mass, ω = angular speed.

Translational KE = $\frac{1}{2} mv^2$

Rotational KE = $\frac{1}{2} I\omega^2$, where $I = mk^2$

Rolling without slipping: $v = r\omega \Rightarrow \omega = v/r$

Rotational KE = $\frac{1}{2} mk^2(v^2/r^2)$

Total KE $E = \frac{1}{2} mv^2 + \frac{1}{2} mv^2(k^2/r^2)$

$E = \frac{1}{2} mv^2(1 + k^2/r^2)$

Marking: Translational + rotational KE terms – 1 mark · $I = mk^2$ and $v = r\omega$ – 1 mark · Final expression – 1 mark

Examiner tip: Write 'rolling without slipping $\Rightarrow v = r\omega$' explicitly; it carries the middle mark.

Q.16 Water rises to a height of 3 cm in a clean glass capillary tube of radius 0.5 mm. (a) Find the surface tension of water. (b) To what height will it rise in a tube of radius 0.25 mm? (Angle of contact = 0°, density of water = 1000 kg/m^3, g = 9.8 m/s^2)

Answer (3 marks)

Formula: $h = 2T\cos\theta/(r\rho g)$; with $\theta = 0°$, $T = hr\rho g/2$

(a) $T = (3 \times 10^{-2} \times 0.5 \times 10^{-3} \times 1000 \times 9.8)/2$

$= (14.7 \times 10^{-2})/2 =$ **7.35×10^{-2} N/m**

(b) $h \propto 1/r$ (same liquid and glass), so $h_2 = h_1(r_1/r_2) = 3 \times (0.5/0.25) =$ **6 cm**

Marking: Formula $h = 2T\cos\theta/r\rho g$ – 1 mark · $T = 7.35 \times 10^{-2}$ N/m – 1 mark · $h_2 = 6$ cm – 1 mark

Examiner tip: Use radius, not diameter, and convert cm and mm to metres before substituting.

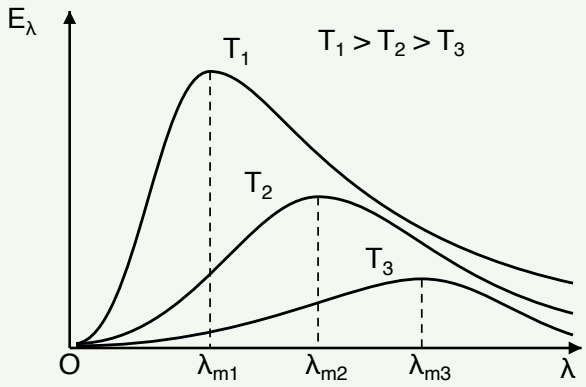
Q.17 State Wien's displacement law and draw the graphs of spectral emissive power against wavelength for a perfectly black body at three different temperatures. The radiation from a star has maximum intensity at 580 nm. Estimate the temperature of its surface. ($b = 2.9 \times 10^{-3}$ m K)

Answer (3 marks)

Wien's displacement law: The wavelength λ_m at which a perfectly black body emits maximum energy is inversely proportional to its absolute temperature: $\lambda_m T = b$ (constant).

Graph: see figure – as T increases, the peak of the curve shifts towards shorter wavelength and the area under the curve (total energy) increases.

Numerical: $T = b/\lambda_m = (2.9 \times 10^{-3})/(580 \times 10^{-9}) =$ **5000 K**



Marking: Statement – 1 mark · Labelled graph with peaks shifting to shorter λ at higher T – 1 mark · T = 5000 K – 1 mark

Examiner tip: Label both axes and show that the hotter curve lies above everywhere and peaks at a shorter wavelength.

Q.18 A Carnot engine works between a source at 477 °C and a sink at 27 °C. If it absorbs 3500 J of heat from the source in each cycle, find (a) its efficiency, (b) the work done per cycle and (c) the heat rejected to the sink per cycle.

Answer (3 marks)

$T_H = 477 + 273 = 750$ K, $T_C = 27 + 273 = 300$ K, $Q_H = 3500$ J

(a) $\eta = 1 - T_C/T_H = 1 - 300/750 = 1 - 0.4 = 0.6 =$ **60 %**

(b) $W = \eta Q_H = 0.6 \times 3500 =$ **2100 J**

(c) $Q_C = Q_H - W = 3500 - 2100 =$ **1400 J**

Marking: Efficiency 60 % (temperatures in K) – 1 mark · W = 2100 J – 1 mark · Q_C = 1400 J – 1 mark

Examiner tip: Temperatures must be in kelvin; using 27/477 gives a wrong efficiency and zero marks for (a).

Q.19 Show that, for small amplitudes, the motion of a simple pendulum is linear S.H.M. Hence obtain the expression for its period.

Answer (3 marks)

Let a bob of mass m hang by a string of length L. When displaced through a small angle θ (arc OP = x), the forces on the bob are its weight mg and the tension T (see figure).

mg is resolved into mg cos θ along the string (balanced by T) and mg sin θ perpendicular to the string, towards the mean position O.

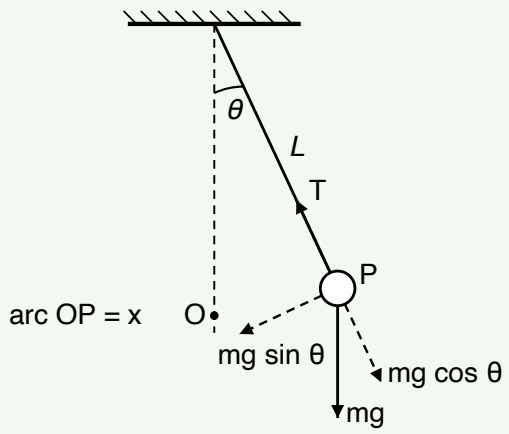
Restoring force $F = -mg \sin\theta$. For small θ, $\sin\theta \approx \theta = x/L$

$\therefore F = -mg\,x/L \Rightarrow$ acceleration $a = F/m = -(g/L)\,x$

Since g/L is constant, $a \propto -x$: acceleration is proportional to displacement and directed towards the mean position, so the motion is linear S.H.M.

Comparing with $a = -\omega^2 x$: $\omega^2 = g/L \Rightarrow \omega = \sqrt{(g/L)}$

Period $\mathbf{T = 2\pi/\omega = 2\pi\sqrt{(L/g)}}$



Marking: Labelled figure with resolved forces – 1 mark · $F = -mg\theta \approx -mgx/L$ and $a \propto -x \Rightarrow$ S.H.M. – 1 mark · $T = 2\pi\sqrt{(L/g)}$ – 1 mark

Examiner tip: Say clearly 'for small θ, sin θ ≈ θ' – without it the motion is not S.H.M.

Q.20 What are beats? Two tuning forks A and B produce 5 beats per second when sounded together. The frequency of A is 320 Hz. When A is filed slightly, the number of beats per second decreases. Find the frequency of B.

Answer (3 marks)

Beats: When two sound waves of slightly different frequencies travelling in the same direction superpose, the intensity of the resultant sound rises and falls periodically. One rise and fall is one beat; beat frequency = $n_1 - n_2$.

Numerical: $n_B = n_A \pm 5 = 325$ Hz or 315 Hz.

Filing a tuning fork removes mass from its prongs, so its frequency increases.

If $n_B = 315$ Hz, raising n_A would increase the beats. If $n_B = 325$ Hz, raising n_A towards 325 Hz decreases the beats – this agrees with the data.

$\therefore n_B =$ **325 Hz**

Marking: Definition of beats – 1 mark · Two possible values 315/325 Hz – 1 mark · Reasoning (filing raises frequency) and answer 325 Hz – 1 mark

Examiner tip: Write the effect of filing (frequency increases) and of loading with wax (frequency decreases); the answer needs this reasoning.

Q.21 In Young's double-slit experiment, the slits are 0.5 mm apart and the screen is 1 m from the slits. Light of wavelength 600 nm is used. Find (a) the fringe width, (b) the distance of the third bright fringe and (c) the distance of the second dark fringe from the central bright fringe.

Answer (3 marks)

Data: $d = 0.5 \times 10^{-3}$ m, $D = 1$ m, $\lambda = 600 \times 10^{-9}$ m

(a) $W = \lambda D/d = (600 \times 10^{-9} \times 1)/(0.5 \times 10^{-3}) = 1.2 \times 10^{-3}$ m = **1.2 mm**

(b) n-th bright fringe: $y_n = n\lambda D/d = nW \Rightarrow y_3 = 3 \times 1.2 =$ **3.6 mm**

(c) n-th dark fringe: $y'_n = (2n - 1)\lambda D/(2d) = (2n - 1)W/2 \Rightarrow y'_2 = 3 \times 1.2/2 =$ **1.8 mm**

Marking: Fringe width 1.2 mm – 1 mark · Third bright fringe 3.6 mm – 1 mark · Second dark fringe 1.8 mm – 1 mark

Examiner tip: For dark fringes use $(2n - 1)W/2$; students often write $2W$ for the second dark fringe.

Q.22 A parallel plate capacitor has plates of area A separated by a distance d in vacuum. A dielectric slab of dielectric constant k and thickness t (t < d) is placed between the plates as shown. Obtain an expression for its capacitance.

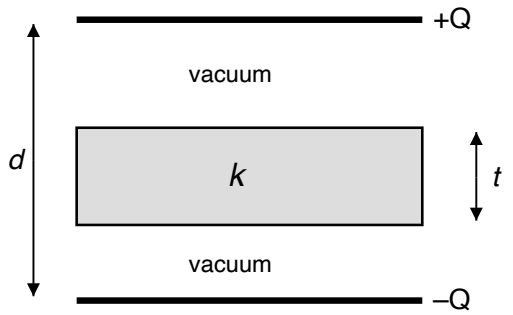


Plate area A; slab thickness t (t < d)

Answer (3 marks)

Let the plates carry charges +Q and –Q; surface charge density $\sigma = Q/A$.

In the vacuum region (total thickness d – t), field $E_0 = \sigma/\varepsilon_0 = Q/(\varepsilon_0 A)$.

Inside the slab the field is reduced by k: $E = E_0/k = Q/(k\varepsilon_0 A)$.

Potential difference between the plates:

$V = E_0(d - t) + Et = (Q/\varepsilon_0 A)[(d - t) + t/k]$

Capacitance $C = Q/V$

$C = \varepsilon_0 A / [d - t + t/k] = \varepsilon_0 A / [d - t(1 - 1/k)]$

(Since k > 1, C is greater than the air value $\varepsilon_0 A/d$.)

Marking: Fields $E_0 = \sigma/\varepsilon_0$ in air and E_0/k in slab – 1 mark · $V = E_0(d - t) + E_0 t/k$ – 1 mark · Final expression for C – 1 mark

Examiner tip: Check your result: t = 0 must give $\varepsilon_0 A/d$ and t = d must give $k\varepsilon_0 A/d$.

Q.23 Draw a labelled circuit diagram to determine the internal resistance of a cell using a potentiometer. With the key K_2 open, the balancing length is 300 cm; when the cell is shunted by a resistance of 10 Ω (K_2 closed), the balancing length becomes 250 cm. Find the internal resistance of the cell.

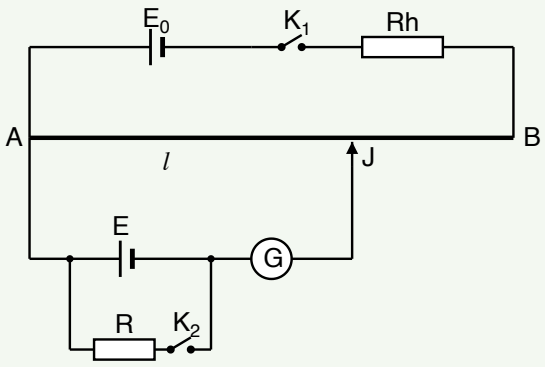
Answer (3 marks)

Circuit: see figure (E_0 – driving cell, K_1 – key, Rh – rheostat, AB – potentiometer wire, E – cell under test, G – galvanometer, J – jockey, R – resistance box with key K_2).

With K_2 open: $E = kl_1$. With K_2 closed: terminal p.d. $V = kl_2$.

$r = R(E - V)/V = R(l_1 - l_2)/l_2$

$r = 10 \times (300 - 250)/250 = 10 \times 50/250 =$ **2 Ω**



Marking: Labelled circuit diagram – 1 mark · Formula $r = R(l_1 - l_2)/l_2$ – 1 mark · $r = 2\ \Omega$ – 1 mark

Examiner tip: Positive terminals of E_0 and E must both be connected to end A, otherwise no null point is obtained.

Q.24 An iron rod of relative permeability 600 is placed inside a solenoid that produces a magnetic intensity of 1000 A/m. Find (a) the magnetic susceptibility of iron, (b) the magnetization of the rod and (c) the magnetic induction in the rod. ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A m, $\pi = 3.14$)

Answer (3 marks)

Data: $\mu_r = 600$, $H = 1000$ A/m

(a) $\chi = \mu_r - 1 =$ **599**

(b) $M = \chi H = 599 \times 1000 =$ **5.99×10^5 A/m**

(c) $B = \mu_0\mu_r H = 4 \times 3.14 \times 10^{-7} \times 600 \times 1000 = 7536 \times 10^{-4}$ T ≈ **0.754 T**

Marking: $\chi = 599$ – 1 mark · $M = 5.99 \times 10^5$ A/m – 1 mark · $B \approx 0.754$ T – 1 mark

Examiner tip: χ is dimensionless; M has the same unit as H (A/m).

Q.25 A series LCR circuit has $R = 40\ \Omega$, $L = (0.8/\pi)$ H and $C = (200/\pi)$ μF. It is connected to a 220 V, 50 Hz a.c. supply. Find (a) the impedance, (b) the r.m.s. current, (c) the power factor and the average power consumed.

Answer (3 marks)

$X_L = 2\pi fL = 2\pi \times 50 \times 0.8/\pi = 80\ \Omega$

$X_C = 1/(2\pi fC) = 1/(2\pi \times 50 \times 200 \times 10^{-6}/\pi) = 1/0.02 = 50\ \Omega$

(a) $Z = \sqrt{[R^2 + (X_L - X_C)^2]} = \sqrt{(40^2 + 30^2)} =$ **50 Ω**

(b) $I_{rms} = e_{rms}/Z = 220/50 =$ **4.4 A**

(c) Power factor $\cos\phi = R/Z = 40/50 =$ **0.8** (current lags the emf since $X_L > X_C$)

Average power $P = e_{rms}I_{rms}\cos\phi = 220 \times 4.4 \times 0.8 =$ **774.4 W** ($= I_{rms}^2 R$)

Marking: X_L, X_C and $Z = 50\ \Omega$ – 1 mark · $I_{rms} = 4.4$ A – 1 mark · $\cos\phi = 0.8$ and $P = 774.4$ W – 1 mark

Examiner tip: Convert μF to F; the π in L and C cancels neatly – look for it before multiplying.

Q.26 Obtain an expression for the de Broglie wavelength of an electron accelerated from rest through a potential difference V. Calculate it for V = 400 V. ($h = 6.63 \times 10^{-34}$ J s, $m = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)

Answer (3 marks)

Work done by the field = gain in kinetic energy: $eV = \tfrac{1}{2}mv^2 = p^2/(2m)$

$\therefore$ momentum $p = \sqrt{(2meV)}$

de Broglie wavelength $\lambda = h/p =$ **$h/\sqrt{(2meV)}$**

For V = 400 V:

$\lambda = 6.63 \times 10^{-34}/\sqrt{(2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 400)}$

$= 6.63 \times 10^{-34}/\sqrt{(1.165 \times 10^{-46})} = 6.63 \times 10^{-34}/(1.079 \times 10^{-23})$

$\lambda =$ **6.14×10^{-11} m ≈ 0.614 Å**

Marking: $eV = \tfrac{1}{2}mv^2 \Rightarrow p = \sqrt{(2meV)}$ – 1 mark · $\lambda = h/\sqrt{(2meV)}$ – 1 mark · $\lambda \approx 6.14 \times 10^{-11}$ m (0.614 Å) – 1 mark

Examiner tip: Use the non-relativistic result only; show the units of λ (Å or m).

SECTION D Attempt any THREE questions of the following: (4 marks each) **[12]**

Q.27 (a) On the basis of the kinetic theory of gases, derive an expression for the pressure exerted by an ideal gas in terms of its density and the r.m.s. speed of its molecules. [3]
(b) The r.m.s. speed of the molecules of a gas at 27 °C is 500 m/s. Find it at 927 °C. [1]

Answer (4 marks)

(a) Consider N molecules, each of mass m, in a cubical box of side L. A molecule with velocity components (v_x, v_y, v_z) hits wall A (perpendicular to x-axis) elastically.

Change in momentum per collision = $-mv_x - mv_x = -2mv_x$; momentum given to the wall = $2mv_x$.

Time between successive collisions with the same wall $\Delta t = 2L/v_x$.

Force on wall by this molecule = $2mv_x/(2L/v_x) = mv_x^2/L$

Total force $F = (m/L)\,\Sigma v_x^2 = (m/L)\,N\langle v_x^2\rangle$

By random motion, $\langle v_x^2\rangle = \langle v_y^2\rangle = \langle v_z^2\rangle = v_{rms}^2/3$

Pressure $P = F/L^2 = (1/3)(Nm/L^3)\,v_{rms}^2$

Since $Nm/L^3 = M/V = \rho$ (density), **$P = (1/3)\,\rho\, v_{rms}^2$**

(b) $v_{rms} \propto \sqrt{T}$; $T_1 = 300$ K, $T_2 = 1200$ K

$v_2 = v_1\sqrt{(T_2/T_1)} = 500 \times \sqrt{4} =$ **1000 m/s**

Marking: Momentum change $2mv_x$ and time $2L/v_x$ – 1 mark · Total force and $\langle v_x^2\rangle = v_{rms}^2/3$ – 1 mark · $P = \tfrac{1}{3}\rho v_{rms}^2$ – 1 mark · (b) 1000 m/s – 1 mark

Examiner tip: State the assumptions you use (elastic collisions, random motion); and in (b) convert °C to K before taking the ratio.

Q.28 (a) State Huygens' principle. Using it, prove the law of refraction of light at a plane surface separating two media. [3]
(b) The refractive index of a glass is $\sqrt{3}$. Find the polarising angle (Brewster's angle) for it and the angle of refraction at this angle of incidence. [1]

Answer (4 marks)

(a) Huygens' principle: (i) Every point on a wavefront acts as a source of secondary wavelets that spread in all directions with the speed of the wave in that medium. (ii) The new wavefront at any later instant is the forward envelope (common tangent) of these secondary wavelets.
Proof (see figure): A plane wavefront AB is incident obliquely on the surface separating medium 1 (speed v_1) and medium 2 (speed v_2), with angle of incidence i. When end A touches the surface, B is still a distance BC from it.

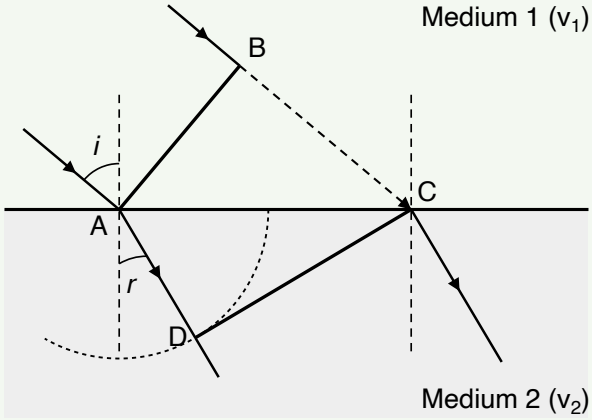
Let light take time t to go from B to C: $BC = v_1 t$. In the same time, the secondary wavelet from A travels into medium 2 a distance $AD = v_2 t$. The tangent CD from C to this wavelet is the refracted wavefront and r is the angle of refraction.

In $\triangle ABC$: $\sin i = BC/AC = v_1 t/AC$; in $\triangle ADC$: $\sin r = AD/AC = v_2 t/AC$

$\therefore \sin i / \sin r = v_1/v_2 =$ constant $= {}_1n_2$ (Snell's law). The incident ray, refracted ray and normal lie in the same plane (the plane of the figure).

(b) Brewster's law: $\tan i_p = n = \sqrt{3} \Rightarrow$ **$i_p = 60°$**

At polarising angle, $i_p + r = 90° \Rightarrow$ **$r = 30°$**



Marking: Statement of Huygens' principle – 1 mark · Labelled wavefront diagram – 1 mark · sin i/sin r = v_1/v_2 derivation – 1 mark · (b) i_p = 60°, r = 30° – 1 mark

Examiner tip: Draw the incident and refracted wavefronts (AB and CD) as well as the rays; a ray diagram alone gets no figure mark.

Q.29 (a) What is self-induction? Obtain an expression for the self-inductance of a long air-cored solenoid. [3]

(b) A solenoid 50 cm long has 500 turns and area of cross-section 10 cm^2. Find its self-inductance. ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A m, $\pi = 3.14$) [1]

Answer (4 marks)

(a) Self-induction: the phenomenon in which a changing current in a coil induces an emf in the same coil, opposing the change of current. Flux linkage $N\Phi = LI$, and $e = -L\,dI/dt$; L is the self-inductance (unit henry).

For a long solenoid of length l, N turns, area A, carrying current I:

$B = \mu_0 nI = \mu_0 NI/l$ (uniform inside)

Flux through one turn $\Phi = BA = \mu_0 NIA/l$

Total flux linkage $N\Phi = \mu_0 N^2 IA/l$

Since $N\Phi = LI$: $\mathbf{L = \mu_0 N^2 A/l}$ ($= \mu_0 n^2 Al$)

(b) $L = (4 \times 3.14 \times 10^{-7} \times 500^2 \times 10 \times 10^{-4})/0.5$

$= (12.56 \times 10^{-7} \times 2.5 \times 10^5 \times 10^{-3})/0.5 = \mathbf{6.28 \times 10^{-4}\ H}$ (≈ 0.628 mH)

Marking: Definition of self-induction – 1 mark · $B = \mu_0 NI/l$ and $N\Phi = \mu_0 N^2 IA/l$ – 1 mark · $L = \mu_0 N^2 A/l$ – 1 mark · (b) 6.28×10^{-4} H – 1 mark

Examiner tip: Area 10 $cm^2 = 10 \times 10^{-4}\ m^2 = 10^{-3}\ m^2$; wrong area conversion is the commonest error.

Q.30 (a) Draw a neat labelled circuit diagram of an n-p-n transistor in common-emitter configuration. Define the current gains α_{dc} and β_{dc} and show that $\beta_{dc} = \alpha_{dc}/(1 - \alpha_{dc})$. [3]

(b) For a transistor $\alpha_{dc} = 0.96$. If the emitter current is 7.5 mA, find β_{dc} and the base current. [1]

Answer (4 marks)

(a) Circuit: see figure (base–emitter junction forward biased by V_{BB}, collector–base junction reverse biased by V_{CC}).

α_{dc} (common-base d.c. current gain) = I_C/I_E

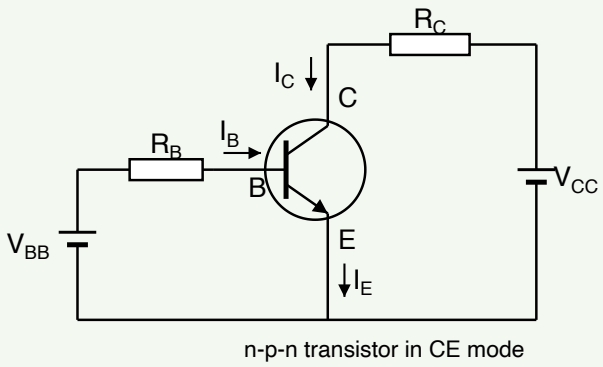
β_{dc} (common-emitter d.c. current gain) = I_C/I_B

Relation: $I_E = I_B + I_C$. Dividing by I_C: $I_E/I_C = I_B/I_C + 1$

$1/\alpha_{dc} = 1/\beta_{dc} + 1 \Rightarrow 1/\beta_{dc} = (1 - \alpha_{dc})/\alpha_{dc} \Rightarrow \mathbf{\beta_{dc} = \alpha_{dc}/(1 - \alpha_{dc})}$

(b) $\beta_{dc} = 0.96/(1 - 0.96) = 0.96/0.04 =$ **24**

$I_C = \alpha_{dc} I_E = 0.96 \times 7.5 = 7.2$ mA $\Rightarrow I_B = I_E - I_C = 7.5 - 7.2 =$ **0.3 mA**



n-p-n transistor in CE mode

Marking: Labelled CE circuit with correct biasing – 1 mark · Definitions of α_{dc} and β_{dc} – 1 mark · Derivation $\beta_{dc} = \alpha_{dc}/(1 - \alpha_{dc})$ – 1 mark · (b) $\beta_{dc} = 24$, $I_B = 0.3$ mA – 1 mark

Examiner tip: The emitter arrow of an n-p-n transistor points OUT of the transistor; draw it correctly and show biasing polarities.

Q.31 (a) State the law of radioactive decay. Derive the relation $N = N_0 e^{-\lambda t}$ and hence obtain the relation between half-life and decay constant. [3]

(b) A radioactive sample has a half-life of 20 minutes and an initial activity of 6.4×10^4 Bq. Find its activity after 1 hour. [1]

Answer (4 marks)

(a) Law: The rate of decay of radioactive nuclei at any instant is proportional to the number of undecayed nuclei present at that instant: $dN/dt = -\lambda N$, where λ is the decay constant.

Derivation: $dN/N = -\lambda\, dt$. Integrating, $\ln N = -\lambda t + C$. At $t = 0$, $N = N_0 \Rightarrow C = \ln N_0$.

$\ln(N/N_0) = -\lambda t \Rightarrow$ **$N = N_0 e^{-\lambda t}$**

Half-life $T_{1/2}$: at $t = T_{1/2}$, $N = N_0/2 \Rightarrow 1/2 = e^{-\lambda T_{1/2}} \Rightarrow \lambda T_{1/2} = \ln 2$

$T_{1/2} = 0.693/\lambda$

(b) 1 hour = 60 min = 3 half-lives. Activity $\propto N$, so $A = A_0(1/2)^3 = 6.4 \times 10^4/8 =$ **8×10^3 Bq**

Marking: Statement $dN/dt = -\lambda N$ – 1 mark · Integration to $N = N_0 e^{-\lambda t}$ – 1 mark · $T_{1/2} = 0.693/\lambda$ – 1 mark · (b) 8×10^3 Bq – 1 mark

Examiner tip: Show the limits/constant of integration; writing the final result without integrating gets only the statement mark.

All the best for your exam!