Mathematics and Statistics (Arts and Science)

HSC Board Exam February–March 2027 – Practice Paper No. 2

Practice Question Paper

Time: 3 hours **Max. Marks: 80**

(1) The question paper is divided into FOUR sections.
(2) Section A: Q.1 contains EIGHT multiple choice type questions, each carrying TWO marks. Q.2 contains FOUR very short answer type questions, each carrying ONE mark.
(3) Section B: Q.3 to Q.14 contain TWELVE short answer type questions, each carrying TWO marks. Attempt any EIGHT.
(4) Section C: Q.15 to Q.26 contain TWELVE short answer type questions, each carrying THREE marks. Attempt any EIGHT.
(5) Section D: Q.27 to Q.34 contain EIGHT long answer type questions, each carrying FOUR marks. Attempt any FIVE.
(6) Use of log table is allowed. Use of calculator is not allowed.
(7) Figures to the right indicate full marks.
(8) Use of graph paper is not necessary. Only a rough sketch of the graph is expected.
(9) For each MCQ, write the correct answer along with its alphabet, e.g. (a) ..... / (b) ..... etc. Only the first attempt of each MCQ will be evaluated.
(10) Start the answer to each section on a new page.

SECTION A – Q.1 Select and write the correct answer for the following multiple choice type of questions. **[16]**

(i) Which of the following statement patterns is a tautology?

(a) $q \rightarrow (p \wedge q)$ (b) $(p \wedge \sim q) \rightarrow p$
(c) $(p \vee q) \rightarrow p$ (d) $p \wedge (p \rightarrow \sim p)$

(ii) If $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ and the elementary transformation $R_2 \rightarrow R_2 - 2R_1$ is applied to A, then the second row of the new matrix is .....

(a) [0 5] (b) [0 1]
(c) [8 1] (d) [0 −5]

(iii) The value of $\cos[2 \sin^{-1}(1/3)]$ is .....

(a) $4\sqrt{2}/9$ (b) −7/9
(c) 7/9 (d) 2/3

(iv) The sum of the slopes of the lines represented by $3x^2 + 8xy - 5y^2 = 0$ is .....

(a) 8/5 (b) −8/5
(c) 3/5 (d) −3/5

(v) If the vectors $\bar{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\bar{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular, then λ = ...

(a) −5/2 (b) 3/2
(c) 5/2 (d) 1

(vi) The point on the curve $y = x^2 - 6x + 11$ at which the tangent is parallel to the X-axis is .....

(a) (2, 3) (b) (3, 2)
(c) (−3, 38) (d) (3, −2)

(vii) $\int \cot x \cdot \log(\sin x)\, dx$ = ...

(a) $(1/2)[\log(\sin x)]^2 + c$ (b) $\log[\log(\sin x)] + c$

(c) $[\log(\sin x)]^2 + c$ (d) $(1/2)\log(\sin x) + c$

(viii) The p.m.f. of a discrete random variable X is $P(X = x) = kx^2$ for x = 1, 2, 3 and 0 otherwise. Then $P(X \geq 2)$ = ...

(a) 1/14 (b) 9/14

(c) 2/7 (d) 13/14

Q.2 Answer the following questions. **[4]**

(i) Find the principal solutions of $2\cos\theta + \sqrt{3} = 0$.

(ii) Write the vector equation of the line passing through the point (2, −1, 3) and parallel to the vector $4\hat{i} - \hat{j} + 2\hat{k}$.

(iii) Find the order and degree of the differential equation $(d^2y/dx^2)^3 + 4(dy/dx)^5 - 7y = \sin x$.

(iv) If X ~ B(12, 1/4), find E(X) and Var(X).

SECTION B Attempt any EIGHT of the following questions (Q.3 to Q.14, 2 marks each). **[16]**

Q.3 Using the rules of negation, write the negation of $p \rightarrow (q \wedge \sim r)$ and simplify it.

Q.4 Find the inverse of $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ by elementary row transformations.

Q.5 Find the value of $\cos[\pi/4 + \tan^{-1}(-1/\sqrt{3})]$.

Q.6 Find the separate equations of the lines represented by $10x^2 + 9xy - 9y^2 = 0$.

Q.7 Using vectors, show that the points A(2, −1, 3), B(4, 3, 1) and C(3, 1, 2) are collinear.

Q.8 The corner points of the feasible region of an L.P.P. are O(0, 0), A(6, 0), B(4, 3) and C(0, 5). Find the maximum value of z = 4x + 5y and the point at which it occurs.

Q.9 If $x = 4\sec\theta$ and $y = 3\tan\theta$, find dy/dx. Also find its value at $\theta = \pi/6$.

Q.10 Evaluate: $\int_0^{\log 2} e^x / [(e^x + 1)(e^x + 2)]\, dx$

Q.11 Complete the following activity to find the area of the region bounded by the curve $y = x^2 + 2$, the X-axis and the lines x = 0 and x = 3.

Activity: For $0 \leq x \leq 3$, $y = x^2 + 2 > 0$, so the region lies above the X-axis.

Required area = $\int_{\square}^{3} (x^2 + 2)\, dx = [\square]_0^3 = 9 + \square = \square$ sq. units

Q.12 Solve the differential equation $(x + 2)\, dy/dx = y + 1$, given that y = 3 when x = 0.

Q.13 A grocer has a tray of 6 eggs, of which 2 are cracked. Two eggs are picked at random without replacement. If X denotes the number of cracked eggs picked, find the probability distribution of X.

Q.14 A fair die is thrown 4 times. Getting a multiple of 3 is a success. Find the probability of exactly 2 successes.

SECTION C Attempt any EIGHT of the following questions (Q.15 to Q.26, 3 marks each). **[24]**

Q.15 Using a truth table, show that $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautology.

Q.16 Find the co-ordinates of the foot of the perpendicular drawn from the point P(7, 5, 5) to the line $(x - 1)/2 = (y - 2)/3 = (z - 3)/4$. Also find the length of the perpendicular.

Q.17 Two straight roads leave a village square C at an angle of 60°. Village B is 8 km from C along one road and village A is 15 km from C along the other. Taking △ABC with a = 8 km, b = 15 km and C = 60°, find (a) the distance AB = c (b) the area of △ABC (c) cos A.

Q.18 Find the joint equation of the pair of lines passing through the point (1, −2) and parallel to the lines represented by $2x^2 - 5xy + 2y^2 = 0$.

Q.19 A and B are the points with position vectors $\hat{i} + 4\hat{j} - 2\hat{k}$ and $5\hat{i} - 4\hat{j} + 6\hat{k}$. Find the position vectors of the points which divide the segment AB (a) internally (b) externally in the ratio 3 : 1.

Q.20 Find the vector and Cartesian equations of the plane passing through the point (1, −2, 3) and perpendicular to each of the planes $2x + y - z = 4$ and $x - y + 2z = 1$.

Q.21 Solve the following L.P.P. graphically:
Minimise $z = 3x + 5y$ subject to $2x + y \geq 8$, $x + 2y \geq 7$, $x \geq 0$, $y \geq 0$.

Q.22 If $y = (2x + 1)^3 \sqrt{(x - 1) / (x + 4)^2}$, find dy/dx.

Q.23 Find the equations of the tangent and the normal to the curve $x^2 + xy + 2y^2 = 4$ at the point (1, 1).

Q.24 Complete the following activity to evaluate $\int dx/(5 + 3\cos x)$.
Activity: Put $\tan(x/2) = t$, so $dx =$ ☐ and $\cos x = (1 - t^2)/(1 + t^2)$.
$5 + 3\cos x = [5(1 + t^2) + 3(1 - t^2)]/(1 + t^2) =$ ☐$/(1 + t^2)$
$I = \int 2\,dt/(8 + 2t^2) = \int dt/($☐$ + t^2) = (1/2)\tan^{-1}($☐$) + c$
$\therefore I =$ ☐ $+ c$

Q.25 Find the area of the region enclosed between the parabolas $y = x^2$ and $y = 8 - x^2$.

Q.26 Solve the differential equation $x\,dy/dx = y + 2x\,e^{-y/x}$, given that $y = 0$ when $x = 1$.

SECTION D Attempt any FIVE of the following questions (Q.27 to Q.34, 4 marks each). **[20]**

Q.27 Solve the following equations by the method of inversion:
$x - y + z = 2$, $2x + y - z = 1$, $x + 2y + z = 11$

Q.28 Find the general solution of $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$. Hence write its principal solutions.

Q.29 A(1, 0, 2), B(3, 1, 1), C(2, 3, 4) and D(4, 2, 5) are four points in space. Find (a) the area of △ABC (b) the volume of the tetrahedron ABCD.

Q.30 A farmer wants to fence a rectangular cattle pen of area 800 m^2 next to the straight stone wall of his farmhouse. No fencing is needed along the wall. Find the dimensions of the pen for which the length of fencing is least. Also find the cost of this fencing at ₹ 150 per metre.

Q.31 Complete the following activity to evaluate $I = \int_0^{\pi/4} x\tan^2 x\,dx$.
Activity: $\tan^2 x =$ ☐ $- 1$, so $I = \int_0^{\pi/4} x\sec^2 x\,dx - \int_0^{\pi/4} x\,dx$
Integrating by parts: $\int x\sec^2 x\,dx = x\tan x - \int$ ☐ $dx = x\tan x + \log(\cos x) + c$
$\therefore \int_0^{\pi/4} x\sec^2 x\,dx = [x\tan x + \log(\cos x)]_0^{\pi/4} = \pi/4 +$ ☐
$\int_0^{\pi/4} x\,dx = [x^2/2]_0^{\pi/4} =$ ☐
$\therefore I =$ ☐

Q.32 Evaluate: $\int (3x + 2)/(x^2 + 4x + 13)\,dx$

Q.33 A cup of tea at 90 °C is kept in a room whose temperature is 26 °C. It cools to 58 °C in 10 minutes. Using Newton's law of cooling, find (a) the temperature of the tea after 20 minutes (b) the time it takes to cool to 34 °C.

Q.34 The p.d.f. of a continuous random variable X is $f(x) = k(4x - x^2)$ for $0 \leq x \leq 4$, and $f(x) = 0$ otherwise. Find (a) k (b) the c.d.f. $F(x)$ for $0 \leq x \leq 4$ (c) $P(1 < X < 3)$.

All the best for your exam!