Mathematics (Standard) – Code 041

CBSE Class 10 Board Examination 2027 – Practice Paper No. 1

Model Answer Paper

Time: 3 hours **Max. Marks: 80**

Model answers show one correct method; any other correct method gets full marks.

SECTION A This section comprises 18 multiple choice questions (Q.1 to Q.18) of 1 mark each. Choose the correct option. **[18]**

1. If $p = 2^3 \times 3^2 \times 5$ and $q = 2^2 \times 3^3 \times 7$, then LCM(p, q) ÷ HCF(p, q) is:

(A) 70 (B) 105

(C) 210 (D) 420

Answer (1 mark)

(C) 210

HCF = $2^2 \times 3^2 = 36$ (smallest powers of common primes); LCM = $2^3 \times 3^3 \times 5 \times 7 = 7560$ (greatest powers of all primes). $7560 \div 36 = 210 = 2 \times 3 \times 5 \times 7$.

2. The graph of y = p(x), where p(x) is a quadratic polynomial, cuts the x-axis at (−3, 0) and (5, 0) and the y-axis at (0, −30). Then p(x) is:

(A) $x^2 - 2x - 15$ (B) $2x^2 - 4x - 30$

(C) $2x^2 + 4x - 30$ (D) $-2x^2 + 4x + 30$

Answer (1 mark)

(B) $2x^2 - 4x - 30$

Zeroes −3 and 5 ⇒ $p(x) = k(x + 3)(x - 5)$. $p(0) = -15k = -30 \Rightarrow k = 2$. So $p(x) = 2(x^2 - 2x - 15) = 2x^2 - 4x - 30$.

3. The value of k for which the pair of linear equations 3x + ky = 7 and 6x + 10y = 11 has no solution is:

(A) 5 (B) −5

(C) 10 (D) 20

Answer (1 mark)

(A) 5

No solution ⇒ $a_1/a_2 = b_1/b_2 \neq c_1/c_2$: $3/6 = k/10 \Rightarrow k = 5$, and $3/6 = 1/2 \neq 7/11$ ✓.

4. If the quadratic equation $4x^2 - kx + 9 = 0$ has two equal real roots and k > 0, then the value of k is:

(A) 6 (B) 9

(C) 36 (D) 12

Answer (1 mark)

(D) 12

Equal roots ⇒ discriminant $b^2 - 4ac = 0$: $k^2 - 4 \times 4 \times 9 = 0 \Rightarrow k^2 = 144 \Rightarrow k = 12$ (k > 0).

5. The 15th term from the end of the A.P. 7, 11, 15, ..., 187 is:

(A) 127 (B) 131

(C) 135 (D) 63

Answer (1 mark)
(B) 131
From the end, the A.P. is 187, 183, 179, ... with first term 187 and d = −4. 15th term = 187 + 14 × (−4) = 187 − 56 = 131.

6. In △PQR, S and T are points on the sides PQ and PR respectively such that ST ∥ QR. If PS = 4 cm, SQ = 6 cm and PR = 15 cm, then PT is:

(A) 6 cm (B) 7.5 cm
(C) 9 cm (D) 10 cm

Answer (1 mark)
(A) 6 cm
By the Basic Proportionality Theorem, PS/PQ = PT/PR ⇒ 4/10 = PT/15 ⇒ PT = 6 cm.

7. The point on the y-axis which is equidistant from the points A(5, −2) and B(−3, 2) is:

(A) (0, 2) (B) (0, −2)
(C) (−2, 0) (D) (2, 0)

Answer (1 mark)
(B) (0, −2)
Let the point be (0, y). $(0-5)^2 + (y+2)^2 = (0+3)^2 + (y-2)^2 \Rightarrow 25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4 \Rightarrow 8y = -16 \Rightarrow y = -2$. (Check: both squared distances are 25.)

8. If sin A = 8/17, where A is an acute angle, then the value of cot A + cosec A is:

(A) 4 (B) 23/8
(C) 25/8 (D) 15/17

Answer (1 mark)
(A) 4
Take opposite side 8k, hypotenuse 17k ⇒ adjacent side = $\sqrt{(289-64)}\,k = 15k$. cot A = 15/8, cosec A = 17/8 ⇒ sum = 32/8 = 4.

9. A number is selected at random from the numbers 1, 2, 3, ..., 50. The probability that it is divisible by both 3 and 4 is:

(A) 2/25 (B) 12/25
(C) 1/12 (D) 7/25

Answer (1 mark)
(A) 2/25
Divisible by both 3 and 4 ⇔ divisible by 12: 12, 24, 36, 48 → 4 favourable outcomes out of 50. P = 4/50 = 2/25.

10. A circular pizza of diameter 28 cm is cut into 8 equal slices by cuts through its centre. The area of the top surface of one slice is:

(A) 38.5 cm^2 (B) 77 cm^2
(C) 88 cm^2 (D) 154 cm^2

Answer (1 mark)
(B) 77 cm^2
Each slice is a sector of angle 360°/8 = 45° and radius 14 cm. Area = (1/8) × (22/7) × 14 × 14 = 616/8 = 77 cm^2.

11. Two identical solid hemispheres, each of radius 3.5 cm, are joined together along their flat faces. The total surface area of the solid so formed is:

(A) 77 cm^2 (B) 115.5 cm^2

(C) 154 cm^2 (D) 231 cm^2

Answer (1 mark)
(C) 154 cm^2
The flat faces are hidden, so only the two curved surfaces remain: $2 \times 2\pi r^2 = 4 \times (22/7) \times 3.5 \times 3.5 = 154$ cm^2 (the solid is a sphere).

12. While finding the mean of a grouped frequency distribution by the assumed mean method, a student took the assumed mean $a = 47.5$ and obtained $\Sigma f_i = 40$ and $\Sigma f_i d_i = -60$, where $d_i = x_i - a$. The mean of the data is:

(A) 45.5 (B) 46

(C) 46.5 (D) 49

Answer (1 mark)
(B) 46
Mean $= a + \Sigma f_i d_i / \Sigma f_i = 47.5 + (-60/40) = 47.5 - 1.5 = 46$.

13. One letter is chosen at random from the letters of the word MAHARASHTRA. The probability that the chosen letter is A is:

(A) 4/11 (B) 3/11

(C) 7/11 (D) 4/7

Answer (1 mark)
(A) 4/11
MAHARASHTRA has 11 letters, of which 4 are A. $P(A) = 4/11$.

14. The coordinates of the point which divides the line segment joining A(2, −5) and B(−4, 7) internally in the ratio 1 : 2 are:

(A) (0, −1) (B) (−2, 3)

(C) (−1, 1) (D) (0, 1)

Answer (1 mark)
(A) (0, −1)
By the section formula with $m_1 : m_2 = 1 : 2$: $x = (1 \times (-4) + 2 \times 2)/3 = 0$, $y = (1 \times 7 + 2 \times (-5))/3 = -1$.

15. The area of a sector of a circle of radius 6 cm is 6π cm^2. The angle of the sector is:

(A) 30° (B) 45°

(C) 60° (D) 90°

Answer (1 mark)
(C) 60°
$(\theta/360°) \times \pi \times 6^2 = 6\pi \Rightarrow \theta = 6 \times 360°/36 = 60°$.

16. For a moderately skewed distribution, the mode is 24 and the mean is 30. Using the empirical relationship between the three measures of central tendency, the median is:

(A) 26 (B) 27

(C) 28 (D) 29

Answer (1 mark)
(C) 28
3 Median = Mode + 2 Mean = 24 + 60 = 84 ⇒ Median = 28.

17. A drone is hovering vertically above a point A on level ground. From a point B on the ground, 45 m away from A, the angle of elevation of the drone is 60°. The height of the drone above the ground is:

(A) $15\sqrt{3}$ m
(B) 45 m
(C) 90 m
(D) $45\sqrt{3}$ m

Answer (1 mark)
(D) $45\sqrt{3}$ m
$\tan 60° = h/45 \Rightarrow h = 45\sqrt{3}$ m.

18. A bag contains 7 red balls, 9 blue balls and some green balls. If the probability of drawing a blue ball at random is 3/8, then the number of green balls in the bag is:

(A) 6
(B) 8
(C) 9
(D) 24

Answer (1 mark)
(B) 8
Let total balls = T. $9/T = 3/8 \Rightarrow T = 24$. Green balls = 24 − 7 − 9 = 8.

Q.19–20 Assertion–Reason based questions. Directions: In Question numbers 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Select the correct answer from the options (A), (B), (C) and (D) given below each question. (1 mark each) **[2]**

19. **Assertion (A):** The points A(1, 2), B(4, 6) and C(7, 10) are collinear.
Reason (R): If for three points A, B and C, AB + BC = AC, then the points A, B and C are collinear.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Assertion (A) is false but Reason (R) is true.

Answer (1 mark)
(A) Both A and R are true and R is the correct explanation of A.
$AB = \sqrt{(3^2 + 4^2)} = 5$, $BC = \sqrt{(3^2 + 4^2)} = 5$, $AC = \sqrt{(6^2 + 8^2)} = 10$. AB + BC = AC, so by R the points are collinear.

20. **Assertion (A):** The quadratic equation $3x^2 - 4x + 2 = 0$ has two distinct real roots.
Reason (R): A quadratic equation $ax^2 + bx + c = 0$ has two distinct real roots if $b^2 - 4ac > 0$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Assertion (A) is false but Reason (R) is true.

Answer (1 mark)
(D) A is false but R is true.
For $3x^2 - 4x + 2 = 0$, $b^2 - 4ac = 16 - 24 = -8 < 0$, so it has no real roots; A is false. R is the correct condition for two distinct real roots.

SECTION B This section comprises 5 very short answer (VSA) type questions of 2 marks each. **[10]**

21. (A) A florist in Pune has 132 roses and 198 marigolds. She wants to make identical bouquets using all the flowers, with each bouquet having the same number of roses and the same number of marigolds. Using prime factorisation, find the greatest number of bouquets she can make. How many roses and marigolds will each bouquet have?

OR

(B) When the students of a school are made to stand in rows of 18, 24 or 30, exactly 5 students are left over each time. Using prime factorisation, find the least possible number of students in the school.

Answer (2 marks)

(A) $132 = 2^2 \times 3 \times 11$; $198 = 2 \times 3^2 \times 11$.

HCF(132, 198) = $2 \times 3 \times 11 = 66$.

Greatest number of bouquets = 66.

Each bouquet: $132 \div 66 = 2$ roses and $198 \div 66 = 3$ marigolds.

OR

(B) $18 = 2 \times 3^2$; $24 = 2^3 \times 3$; $30 = 2 \times 3 \times 5$.

LCM(18, 24, 30) = $2^3 \times 3^2 \times 5 = 360$.

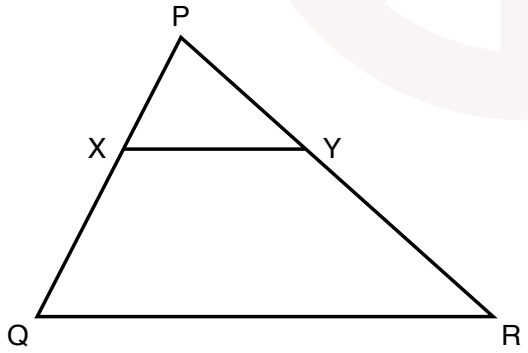
The number leaves remainder 5 on division by each, so it is 360 + 5.

Least possible number of students = 365.

Marking: (A) Prime factorisation of both numbers – ½ mark · HCF = 66 (greatest number of bouquets) – 1 mark · 2 roses and 3 marigolds – ½ mark · (B) Prime factorisation of 18, 24, 30 – ½ mark · LCM = 360 – 1 mark · Adding the remainder: 365 – ½ mark

Examiner tip: Read whether the situation needs HCF (largest equal group) or LCM (smallest common multiple). In (B), many students forget to add the remainder 5.

22. (A) In the given figure, X and Y are points on the sides PQ and PR of △PQR such that XY ∥ QR. If PX = 3 cm, XQ = 4.5 cm and YR = 6 cm, find PY and PR.



OR

(B) PA and PB are tangents drawn from an external point P to a circle with centre O and radius 5 cm. If OP = 13 cm, find the length of the tangent PA and the perimeter of the quadrilateral OAPB.

Answer (2 marks)

(A) In △PQR, XY ∥ QR, so by the Basic Proportionality Theorem,

PX/XQ = PY/YR

⇒ 3/4.5 = PY/6

⇒ PY = (3 × 6)/4.5 = 4 cm.

PR = PY + YR = 4 + 6 = 10 cm.

OR

(B) The tangent at any point of a circle is perpendicular to the radius through the point of contact, so $\angle OAP = 90°$.

In right $\triangle OAP$, $PA^2 = OP^2 - OA^2 = 169 - 25 = 144 \Rightarrow PA = 12$ cm.

Tangents from an external point are equal, so PB = PA = 12 cm; OB = OA = 5 cm.

Perimeter of OAPB = 5 + 12 + 12 + 5 = 34 cm.

Marking: (A) Writing PX/XQ = PY/YR by BPT – ½ mark · PY = 4 cm – 1 mark · PR = 10 cm – ½ mark · (B) $\angle OAP = 90°$ with reason – ½ mark · PA = 12 cm – 1 mark · Perimeter = 34 cm (using PB = PA) – ½ mark

Examiner tip: In (A), PR is the whole side, not PY. In (B), state the reason $\angle OAP = 90°$ (tangent $\perp$ radius) before using Pythagoras.

23. If $7 \sec\theta = 25$, where θ is an acute angle, find the value of $(\sin\theta - \cos\theta)/(\sin\theta + \cos\theta)$.

Answer (2 marks)

$\sec\theta = 25/7 \Rightarrow \cos\theta = 7/25$.

$\sin\theta = \sqrt{(1 - \cos^2\theta)} = \sqrt{(1 - 49/625)} = \sqrt{(576/625)} = 24/25$.

$(\sin\theta - \cos\theta)/(\sin\theta + \cos\theta) = (24/25 - 7/25)/(24/25 + 7/25) = (17/25)/(31/25) = 17/31$.

Marking: $\cos\theta = 7/25$ – ½ mark · $\sin\theta = 24/25$ – ½ mark · Substitution and answer 17/31 – 1 mark

Examiner tip: $\sec\theta$ is the reciprocal of $\cos\theta$, not of $\sin\theta$. Keep the common denominator 25 to avoid arithmetic slips.

24. A circular rangoli of radius 14 cm with centre O is drawn on the floor of a school hall. A square ABCD is marked inside it with all four vertices on the circle, so that each side of the square subtends a right angle at O. The four segments of the circle lying outside the square are filled with white colour. Find the total area of the region filled with white colour.

Answer (2 marks)

Each side of the square, e.g. AB, cuts off a minor segment of the circle with central angle 90°.

Area of sector OAB = $(90°/360°) \times (22/7) \times 14 \times 14 = (1/4) \times 616 = 154$ cm^2.

$\triangle OAB$ is right-angled at O with OA = OB = 14 cm, so area of $\triangle OAB = (1/2) \times 14 \times 14 = 98$ cm^2.

Area of one minor segment = $154 - 98 = 56$ cm^2.

The four segments are congruent (equal chords subtend equal angles), so the white region = $4 \times 56 =$ **224 cm^2**.

(Check: area of circle − area of square = $616 - 4 \times 98 = 224$ cm^2.)

Marking: Area of sector OAB = 154 cm^2 – ½ mark · Area of $\triangle OAB$ = 98 cm^2 – ½ mark · Area of one segment = 56 cm^2 – ½ mark · Total white area = 224 cm^2 – ½ mark

Examiner tip: Area of segment = area of sector − area of triangle. Since $\angle AOB = 90°$, the radii OA and OB are the base and height of $\triangle OAB$. Do not forget to multiply by 4 for the four segments.

25. Ria and Kabir play a board game in which two dice are thrown together. Ria wins a round if the sum of the numbers on the two dice is a multiple of 5. Kabir wins a round if the two numbers differ by exactly 3. Who has the greater chance of winning a round? Justify by finding both probabilities.

Answer (2 marks)

Total outcomes = $6 \times 6 = 36$.

Sum a multiple of 5 (sum 5 or 10): (1, 4), (2, 3), (3, 2), (4, 1), (4, 6), (5, 5), (6, 4) → 7 outcomes.

P(Ria wins) = 7/36.

Numbers differ by 3: (1, 4), (4, 1), (2, 5), (5, 2), (3, 6), (6, 3) → 6 outcomes. P(Kabir wins) = 6/36 =

1/6.

Since 7/36 > 6/36, Ria has the greater chance of winning a round.

Marking: P(Ria wins) = 7/36 with outcomes listed – 1 mark · P(Kabir wins) = 6/36 with outcomes listed – ½ mark · Correct comparison and conclusion – ½ mark

Examiner tip: List the favourable outcomes; (4, 6) and (6, 4) both count, and (5, 5) is counted once. Compare the probabilities over the same denominator 36.

SECTION C This section comprises 6 short answer (SA) type questions of 3 marks each. **[18]**

26. Given that $\sqrt{2}$ is an irrational number, prove that $(7 - 3\sqrt{2})/5$ is an irrational number.

Answer (3 marks)

Let us assume, to the contrary, that $(7 - 3\sqrt{2})/5$ is rational.

Then $(7 - 3\sqrt{2})/5 = a/b$, where a and b are integers and $b \neq 0$.

$\Rightarrow 7 - 3\sqrt{2} = 5a/b$

$\Rightarrow 3\sqrt{2} = 7 - 5a/b = (7b - 5a)/b$

$\Rightarrow \sqrt{2} = (7b - 5a)/(3b)$.

Since a and b are integers, $(7b - 5a)/(3b)$ is rational, so $\sqrt{2}$ would be rational.

This contradicts the given fact that $\sqrt{2}$ is irrational.

Hence our assumption is wrong, and $(7 - 3\sqrt{2})/5$ is irrational.

Marking: Assuming the number is rational and writing it as a/b ($b \neq 0$) – 1 mark · Expressing $\sqrt{2} = (7b - 5a)/(3b)$ – 1 mark · Contradiction and conclusion – 1 mark

Examiner tip: Write '$b \neq 0$' and state clearly what the contradiction is. Do not just say 'irrational ± rational is irrational' without proof.

27. If α and β are the zeroes of the quadratic polynomial $p(x) = 2x^2 - 3x - 5$, find a quadratic polynomial whose zeroes are $(2\alpha + 1)$ and $(2\beta + 1)$.

Answer (3 marks)

For $p(x) = 2x^2 - 3x - 5$: $\alpha + \beta = -(-3)/2 = 3/2$ and $\alpha\beta = -5/2$.

Sum of new zeroes = $(2\alpha + 1) + (2\beta + 1) = 2(\alpha + \beta) + 2 = 3 + 2 = 5$.

Product of new zeroes = $(2\alpha + 1)(2\beta + 1) = 4\alpha\beta + 2(\alpha + \beta) + 1 = -10 + 3 + 1 = -6$.

Required polynomial = $k[x^2 - (\text{sum})x + \text{product}] = k(x^2 - 5x - 6)$; taking $k = 1$: **$x^2 - 5x - 6$** (or any non-zero multiple of it).

(Check: zeroes of p(x) are 5/2 and −1, so the new zeroes are 6 and −1, and $x^2 - 5x - 6 = (x - 6)(x + 1)$.)

Marking: α + β and αβ – 1 mark · Sum of new zeroes = 5 – ½ mark · Product of new zeroes = −6 – 1 mark · Polynomial $x^2 - 5x - 6$ – ½ mark

Examiner tip: Use $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$ carefully with signs. Any polynomial $k(x^2 - 5x - 6)$, $k \neq 0$, is correct.

28. (A) A housing society in Satara collects old newspapers for recycling. It collects 5 kg in the first week, 8 kg in the second week, 11 kg in the third week, and so on, the collection increasing by 3 kg every week. In how many weeks will the society collect 1025 kg in all? How much newspaper is collected in the last of these weeks?

OR

(B) The 4th term of an A.P. is 18 and its 11th term exceeds three times its 3rd term by 4. Find the A.P. and the sum of its first 20 terms.

Answer (3 marks)

(A) The weekly collections 5, 8, 11, … (in kg) form an A.P. with $a = 5$, $d = 3$.

$S_n = (n/2)[2a + (n - 1)d] = 1025$

$\Rightarrow (n/2)[10 + 3(n - 1)] = 1025 \Rightarrow n(3n + 7) = 2050$

$\Rightarrow 3n^2 + 7n - 2050 = 0 \Rightarrow (n - 25)(3n + 82) = 0$
$\Rightarrow n = 25$ ($n = -82/3$ is rejected, as n is a natural number).
Number of weeks = 25.
Collection in the 25th week = $a_{25} = 5 + 24 \times 3 =$ **77 kg**.

OR

(B) $a_4 = a + 3d = 18$ ...(1)

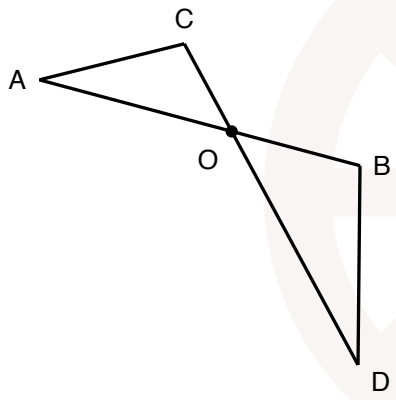
$a_{11} = 3a_3 + 4 \Rightarrow a + 10d = 3(a + 2d) + 4 \Rightarrow 4d - 2a = 4 \Rightarrow 2d - a = 2$ ...(2)

Adding (1) and (2): $5d = 20 \Rightarrow d = 4$, and $a = 18 - 12 = 6$.
A.P.: 6, 10, 14, 18, ...
$S_{20} = (20/2)[2 \times 6 + 19 \times 4] = 10 \times 88 = 880$.

Marking: (A) Forming $S_n = 1025$ – 1 mark · Solving the quadratic, n = 25 – 1 mark · Last week's collection = 77 kg – 1 mark · (B) Two equations in a and d – 1 mark · a = 6, d = 4 and the A.P. – 1 mark · $S_{20} = 880$ – 1 mark

Examiner tip: In (A), reject the negative/fractional value of n with a reason. In (B), translate 'exceeds three times the 3rd term by 4' as $a_{11} = 3a_3 + 4$.

29. In the given figure, the line segments AB and CD intersect at the point O such that OA = 6 cm, OB = 4 cm, OC = 3 cm and OD = 8 cm. Prove that $\triangle AOC \sim \triangle DOB$. If AC = 4.5 cm, find the length of DB.



Answer (3 marks)
In $\triangle AOC$ and $\triangle DOB$:
OA/OD = 6/8 = 3/4 and OC/OB = 3/4, so OA/OD = OC/OB.
$\angle AOC = \angle DOB$ (vertically opposite angles).
Therefore $\triangle AOC \sim \triangle DOB$ (SAS similarity criterion).
Corresponding sides are proportional: AC/DB = OA/OD = 3/4
$\Rightarrow 4.5/DB = 3/4 \Rightarrow DB = (4.5 \times 4)/3 =$ **6 cm**.

Marking: Showing OA/OD = OC/OB – 1 mark · Included angles equal and conclusion by SAS – 1 mark · DB = 6 cm – 1 mark

Examiner tip: For SAS similarity, the equal angle must be the angle included between the two proportional sides. Match vertices in order: A↔D, O↔O, C↔B.

30. Find the ratio in which the point P(m, 6) divides the line segment joining the points A(−1, 3) and B(9, 8). Also find the value of m.

Answer (3 marks)
Let P divide AB in the ratio k : 1.
By the section formula, the y-coordinate of P = (8k + 3)/(k + 1) = 6
$\Rightarrow 8k + 3 = 6k + 6 \Rightarrow 2k = 3 \Rightarrow k = 3/2$.

So the ratio is 3/2 : 1, i.e. **3 : 2** (internally).
x-coordinate: m = (3 × 9 + 2 × (−1))/(3 + 2) = (27 − 2)/5 = 25/5 = 5.
So **m = 5**, and P is (5, 6).

Marking: Using the section formula for the y-coordinate – 1 mark · Ratio 3 : 2 – 1 mark · m = 5 – 1 mark

Examiner tip: Use the coordinate whose value is known (here y = 6) to find the ratio first. Keep the ratio in the order A to B.

31. (A) Sahil was asked to simplify the expression $\tan\theta/(\sec\theta - 1) + \tan\theta/(\sec\theta + 1)$, where θ is acute. He wrote:

$\tan\theta/(\sec\theta - 1) + \tan\theta/(\sec\theta + 1)$

$= \tan\theta[(\sec\theta + 1) + (\sec\theta - 1)] / [(\sec\theta - 1)(\sec\theta + 1)]$ ...(Step 1)

$= 2\tan\theta\sec\theta / (\sec^2\theta - 1)$ ...(Step 2)

$= 2\tan\theta\sec\theta / \tan^2\theta$ ...(Step 3)

$= 2\tan\theta / \sec\theta$ ...(Step 4)

$= 2\sin\theta$ ...(Step 5)

Identify the step in which Sahil made an error, if any. Correct it and find the correct simplified form of the expression in terms of a single trigonometric ratio.

OR

(B) If $\cos\theta - \sin\theta = 1/2$, where θ is an acute angle, find the value of $\tan\theta + \cot\theta$.

Answer (3 marks)

(A) Steps 1 to 3 are correct (in Step 3, $\sec^2\theta - 1 = \tan^2\theta$).

The error is in **Step 4**: $2\tan\theta\sec\theta / \tan^2\theta = 2\sec\theta / \tan\theta$, not $2\tan\theta / \sec\theta$.

Correct working:

$2\sec\theta / \tan\theta = 2 \times (1/\cos\theta) \times (\cos\theta/\sin\theta) = 2/\sin\theta =$ **2 cosec θ**.

OR

(B) Squaring both sides: $\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta = 1/4$

$\Rightarrow 1 - 2\sin\theta\cos\theta = 1/4 \Rightarrow \sin\theta\cos\theta = 3/8$.

$\tan\theta + \cot\theta = \sin\theta/\cos\theta + \cos\theta/\sin\theta = (\sin^2\theta + \cos^2\theta)/(\sin\theta\cos\theta) = 1/(\sin\theta\cos\theta) = 8/3$.

Marking: (A) Identifying Step 4 as the error – 1 mark · Correct form $2\sec\theta/\tan\theta$ – 1 mark · Final answer 2 cosec θ – 1 mark · (B) Squaring and using $\sin^2\theta + \cos^2\theta = 1$ – 1 mark · $\sin\theta\cos\theta = 3/8$ – 1 mark · $\tan\theta + \cot\theta = 8/3$ – 1 mark

Examiner tip: In (A), say which step is wrong and why, then finish the simplification. In (B), write $\tan\theta + \cot\theta$ over the common denominator $\sin\theta\cos\theta$.

SECTION D This section comprises 4 long answer (LA) type questions of 5 marks each. **[20]**

32. A stationery shop owner in Nagpur bought some notebooks for ₹1200. If the price of each notebook had been ₹10 less, he could have bought 10 more notebooks for the same amount. Find the number of notebooks he bought and the price of each notebook.

Answer (5 marks)

Let the number of notebooks bought be x. Then price of each notebook = ₹1200/x.

If the price were ₹10 less, he could buy (x + 10) notebooks:

$1200/x - 10 = 1200/(x + 10)$

$\Rightarrow 1200/x - 1200/(x + 10) = 10$

$\Rightarrow 1200[(x + 10) - x] / [x(x + 10)] = 10$

$\Rightarrow 12000 = 10x(x + 10)$

$\Rightarrow x^2 + 10x - 1200 = 0$

$\Rightarrow x^2 + 40x - 30x - 1200 = 0$
$\Rightarrow (x + 40)(x - 30) = 0$
$\Rightarrow x = 30$ or $x = -40$.
The number of notebooks cannot be negative, so $x = 30$.
Price of each notebook = 1200/30 = ₹40.
Check: at ₹30 each, 1200/30 = 40 notebooks = 30 + 10 ✓.
He bought **30 notebooks at ₹40 each**.

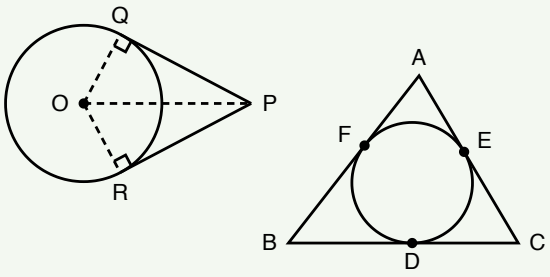
Marking: Forming the equation $1200/x - 10 = 1200/(x + 10)$ – 2 marks · Reducing to $x^2 + 10x - 1200 = 0$ – 1 mark · Solving: $x = 30$ or -40 – 1 mark · Rejecting -40 and stating 30 notebooks at ₹40 – 1 mark

Examiner tip: Define the variable clearly. Give a reason for rejecting the negative root and answer both parts (number and price).

33. Prove that the lengths of the tangents drawn from an external point to a circle are equal. Using the above result, solve the following: In a park, a circular fountain is surrounded by a triangular lawn ABC whose sides AB, BC and CA touch the circular boundary of the fountain at F, D and E respectively. The gardener measures AF = 5 m and BD = 7 m, and the perimeter of the lawn is 36 m. Find the lengths of the three sides of the lawn.

Answer (5 marks)
Given: A circle with centre O; PQ and PR are tangents from an external point P, touching the circle at Q and R.
To prove: PQ = PR.
Construction: Join OP, OQ and OR.
Proof: The tangent at any point of a circle is perpendicular to the radius through the point of contact, so $\angle OQP = \angle ORP = 90°$.
In right triangles OQP and ORP:
OQ = OR (radii of the same circle)
OP = OP (common)
So $\triangle OQP \cong \triangle ORP$ (RHS congruence rule).
Hence PQ = PR (CPCT).

Application: The sides of the lawn are tangents to the circle, and tangents from an external point are equal, so
AE = AF = 5 m, BF = BD = 7 m and CD = CE = z m (say).
Perimeter = AB + BC + CA = (AF + FB) + (BD + DC) + (CE + EA) = (5 + 7) + (7 + z) + (z + 5) = 24 + 2z.
$24 + 2z = 36 \Rightarrow z = 6$.
AB = 5 + 7 = **12 m**, BC = 7 + 6 = **13 m**, CA = 6 + 5 = **11 m**.
(Check: 12 + 13 + 11 = 36 ✓)



Marking: Figure, given, to prove and construction – 1 mark · Proof with $\angle OQP = \angle ORP = 90°$, congruence and conclusion – 2 marks · Equal tangent pairs and perimeter equation – 1 mark · CD = CE = 6 m; sides 12 m, 13 m, 11 m – 1 mark

Examiner tip: In the proof, give the reason for each statement (tangent ⊥ radius, radii equal, RHS). In the application, name the equal tangent pairs (AE = AF, BF = BD, CD = CE) before forming the perimeter equation.

34. (A) A grain silo at a farmers' cooperative in Latur is in the shape of a cylinder of diameter 14 m and height 10 m, surmounted by a conical roof of the same diameter and height 2.4 m. Find (i) the area of metal sheet needed to make the curved surface of the cylinder and the conical roof (the floor is made of concrete), and (ii) the capacity (volume) of the silo.

OR

(B) A solid wooden trophy base is made by fixing a solid cylinder of radius 3.5 cm and height 14 cm centrally on the flat face of a solid hemisphere of radius 10.5 cm. Find (i) the volume of wood used, and (ii) the total surface area of the trophy base.

Answer (5 marks)

(A) r = 7 m, cylinder height h = 10 m, cone height h' = 2.4 m.

Slant height of cone l = $\sqrt{(r^2 + h'^2)} = \sqrt{(49 + 5.76)} = \sqrt{54.76} = 7.4$ m.

(i) Curved surface area of cylinder = $2\pi rh = 2 \times (22/7) \times 7 \times 10 = 440$ m^2.

Curved surface area of cone = $\pi rl = (22/7) \times 7 \times 7.4 = 162.8$ m^2.

Area of metal sheet = 440 + 162.8 = **602.8 m^2**.

(ii) Volume of cylinder = $\pi r^2 h = (22/7) \times 49 \times 10 = 1540$ m^3.

Volume of cone = $(1/3)\pi r^2 h' = (1/3) \times (22/7) \times 49 \times 2.4 = 123.2$ m^3.

Capacity of silo = 1540 + 123.2 = **1663.2 m^3**.

OR

(B) Hemisphere: R = 10.5 cm. Cylinder: r = 3.5 cm, h = 14 cm.

(i) Volume of hemisphere = $(2/3)\pi R^3 = (2/3) \times (22/7) \times 10.5 \times 10.5 \times 10.5 = 2425.5$ cm^3.

Volume of cylinder = $\pi r^2 h = (22/7) \times 3.5 \times 3.5 \times 14 = 539$ cm^3.

Volume of wood = 2425.5 + 539 = **2964.5 cm^3**.

(ii) Curved surface of hemisphere = $2\pi R^2 = 2 \times (22/7) \times 10.5 \times 10.5 = 693$ cm^2.

Exposed part of the flat face (ring) = $\pi(R^2 - r^2) = (22/7) \times (110.25 - 12.25) = (22/7) \times 98 = 308$ cm^2.

Curved surface of cylinder = $2\pi rh = 2 \times (22/7) \times 3.5 \times 14 = 308$ cm^2.

Top of cylinder = $\pi r^2 = (22/7) \times 3.5 \times 3.5 = 38.5$ cm^2.

Total surface area = 693 + 308 + 308 + 38.5 = **1347.5 cm^2**.

Marking: (A) Slant height 7.4 m – 1 mark · CSA of cylinder 440 m^2 – 1 mark · CSA of cone 162.8 m^2 – 1 mark · Total sheet 602.8 m^2 – ½ mark · Capacity 1663.2 m^3 (both volumes) – 1½ marks · (B) Volume of hemisphere – 1 mark · Volume of cylinder – ½ mark · Total volume 2964.5 cm^3 – ½ mark · Curved surface of hemisphere – ½ mark · Exposed ring $\pi(R^2 - r^2)$ – 1 mark · Curved surface and top of cylinder – 1 mark · Total surface area 1347.5 cm^2 – ½ mark

Examiner tip: Include only the surfaces that are actually exposed. In (B), the part of the hemisphere's flat face covered by the cylinder must be subtracted, but the top of the cylinder must be added.

35. (A) A weather balloon is released from a point A on level ground and rises vertically upwards. Vikram, standing at a point O on the same level ground, 120 m from A, observes the balloon at an angle of elevation of 30°. Exactly one minute later, he observes it at an angle of elevation of 60°. Find (i) the height of the balloon at each observation, (ii) the distance between Vikram and the balloon at the second observation, and (iii) the average speed at which the balloon rose, in metres per minute. (Take Vikram's eye level at the ground and use $\sqrt{3} = 1.73$.)

OR

(B) A 6 m long ladder is placed against a vertical wall of a Ganeshotsav pandal so that it makes an angle of 60° with the level ground. The foot of the ladder is then pulled away from the wall along the ground until the ladder makes an angle of 30° with the ground, its top still touching the wall. Find (i) the height reached by the top of the ladder on the wall in each position, (ii) the distance through which the top of the ladder slides down, and (iii) the distance through which the foot of the ladder is moved. (Use $\sqrt{3} = 1.73$)

Answer (5 marks)

(A) Let B_1 and B_2 be the positions of the balloon; OA = 120 m and $\angle OAB_2 = 90°$.

(i) In right $\triangle OAB_1$: $\tan 30° = AB_1/OA \Rightarrow AB_1 = 120 \times (1/\sqrt{3}) = 40\sqrt{3}$ m $\approx 40 \times 1.73 =$ **69.2 m**.

In right $\triangle OAB_2$: $\tan 60° = AB_2/OA \Rightarrow AB_2 = 120\sqrt{3}$ m $\approx$ **207.6 m**.

(ii) $\cos 60° = OA/OB_2 \Rightarrow OB_2 = 120/(1/2) =$ **240 m**.

(iii) Height risen in 1 minute $= 120\sqrt{3} - 40\sqrt{3} = 80\sqrt{3}$ m ≈ 138.4 m.

Average speed = **138.4 m/min** ($80\sqrt{3}$ m/min).

OR

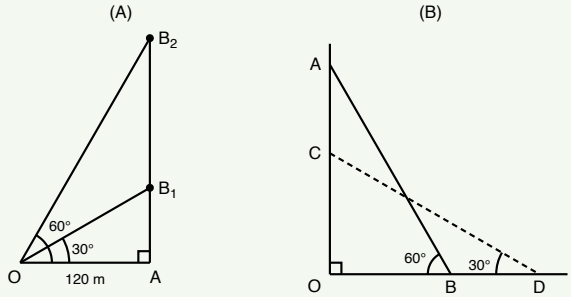
(B) Let O be the foot of the wall. First position of the ladder: AB with $\angle ABO = 60°$; second position: CD with $\angle CDO = 30°$; AB = CD = 6 m and $\angle AOB = 90°$.

(i) In right $\triangle AOB$: $\sin 60° = OA/AB \Rightarrow OA = 6 \times (\sqrt{3}/2) = 3\sqrt{3}$ m $\approx$ **5.19 m**; $\cos 60° = OB/AB \Rightarrow OB = 6 \times (1/2) = 3$ m.

In right $\triangle COD$: $\sin 30° = OC/CD \Rightarrow OC = 6 \times (1/2) =$ **3 m**; $\cos 30° = OD/CD \Rightarrow OD = 6 \times (\sqrt{3}/2) = 3\sqrt{3}$ m.

(ii) Distance the top slides down = $AC = OA - OC = 3\sqrt{3} - 3 = 3(\sqrt{3} - 1) \approx 3 \times 0.73 =$ **2.19 m**.

(iii) Distance the foot moves = $BD = OD - OB = 3\sqrt{3} - 3 \approx$ **2.19 m**.



Marking: Correct figure – 1 mark · (A) First height $40\sqrt{3}$ m ≈ 69.2 m – 1½ marks · Second height $120\sqrt{3}$ m ≈ 207.6 m – 1½ marks · $OB_2 = 240$ m – ½ mark · Average speed $80\sqrt{3} \approx 138.4$ m/min – ½ mark · (B) OA = $3\sqrt{3}$ m and OB = 3 m – 1½ marks · OC = 3 m and OD = $3\sqrt{3}$ m – 1½ marks · Top slides down ≈ 2.19 m – ½ mark · Foot moves ≈ 2.19 m – ½ mark

Examiner tip: Draw and label the figure first; it carries a mark. In (A), the observer stays at O, so both triangles share the base OA = 120 m. In (B), the ladder's length does not change; use sine for the height on the wall and cosine for the distance from the wall.

SECTION E This section comprises 3 case study based questions of 4 marks each. Each case study has three sub-parts (i), (ii) and (iii) of 1, 1 and 2 marks respectively. **[12]**

36. The canteen of a school in Kolhapur sells vada pav and samosas. On Monday it sold 40 vada pav and 30 samosas and collected ₹960. On Tuesday it sold 25 vada pav and 50 samosas and collected ₹975. Let the price of one vada pav be ₹x and the price of one samosa be ₹y.

Based on the above information, answer the following questions:

(i) Form a linear equation in two variables representing Monday's sale. (1 mark)

(ii) Form a linear equation in two variables representing Tuesday's sale. (1 mark)

(iii) (A) Find the price of one vada pav and the price of one samosa. (2 marks)

OR

(iii) (B) On Wednesday, the canteen sold 120 items in all (vada pav and samosas), and the number of vada pav sold was 6 more than the number of samosas sold. Find how many vada pav and how many samosas were sold on Wednesday. (2 marks)

Answer (4 marks)

(i) $40x + 30y = 960$ (or $4x + 3y = 96$).

(ii) $25x + 50y = 975$ (or $x + 2y = 39$).

(iii) (A) From (ii): $x = 39 - 2y$. Substituting in $4x + 3y = 96$:

$4(39 - 2y) + 3y = 96 \Rightarrow 156 - 8y + 3y = 96 \Rightarrow 5y = 60 \Rightarrow y = 12$.

$x = 39 - 24 = 15$.

Price of a vada pav = ₹15; price of a samosa = ₹12.

(Check: $40 \times 15 + 30 \times 12 = 960$ ✓; $25 \times 15 + 50 \times 12 = 975$ ✓.)

OR

(iii) (B) Let p vada pav and q samosas be sold. $p + q = 120$ and $p - q = 6$.

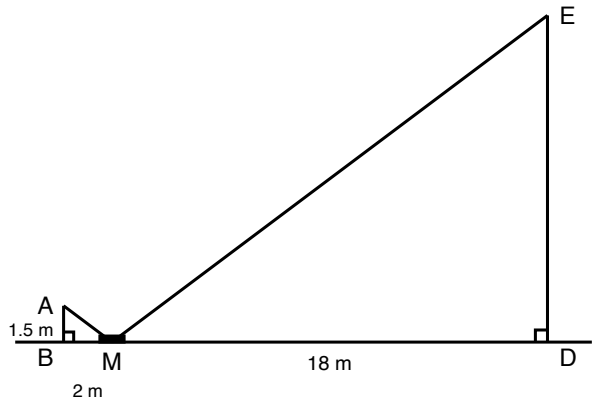
Adding: $2p = 126 \Rightarrow p = 63$; $q = 120 - 63 = 57$.

63 vada pav and 57 samosas were sold on Wednesday.

Marking: (i) Correct equation – 1 mark · (ii) Correct equation – 1 mark · (iii)(A) Solving correctly (substitution/elimination) – 1 mark · x = 15, y = 12 with units – 1 mark · (iii)(B) Forming both equations – 1 mark · 63 vada pav and 57 samosas – 1 mark

Examiner tip: Simplify the equations first (divide by 10 and 25) to make the arithmetic easy, and verify the answer in both original equations.

37. Riya, a Class X student in Nashik, wants to find the height of a tall neem tree without climbing it. She places a small flat mirror M on the level ground and walks back along the line joining the foot of the tree and the mirror until she just sees the top of the tree in the mirror. Her eyes A are 1.5 m above the ground, her feet B are 2 m from the mirror and the mirror is 18 m from the foot D of the tree, whose top is E (see figure). Since the angle of incidence equals the angle of reflection, ∠AMB = ∠EMD.



Based on the above information, answer the following questions:

(i) Name the similarity criterion by which △ABM ~ △EDM. (1 mark)

(ii) Find the height DE of the tree. (1 mark)

(iii) (A) Riya's friend Om, whose eyes are 1.2 m above the ground, uses the same mirror (kept at the same place) to see the top of the same tree. How far from the mirror must Om stand? (2 marks)

OR

(iii) (B) A lamp post 6 m tall stands on the same straight line, on the other side of the mirror from Riya. Standing 2 m from the mirror, Riya just sees the top of the lamp post in the mirror. How far is the foot of the lamp post from the mirror? (2 marks)

Answer (4 marks)

(i) In △ABM and △EDM: ∠ABM = ∠EDM = 90° (Riya and the tree are vertical) and ∠AMB = ∠EMD (given). So △ABM ~ △EDM by the **AA similarity criterion**.

(ii) AB/ED = BM/DM ⇒ 1.5/ED = 2/18 ⇒ ED = (1.5 × 18)/2 = **13.5 m**.

(iii) (A) Let Om stand d m from the mirror. As before, the triangles are similar (AA):
(Om's eye height)/(tree height) = d/18 ⇒ 1.2/13.5 = d/18
⇒ d = (1.2 × 18)/13.5 = 21.6/13.5 = **1.6 m**.

OR

(iii) (B) Let the foot of the lamp post be x m from the mirror. By AA similarity, 1.5/6 = 2/x
⇒ x = (6 × 2)/1.5 = **8 m**.

Marking: (i) AA criterion – 1 mark · (ii) DE = 13.5 m – 1 mark · (iii)(A) Correct proportion from similar triangles – 1 mark · d = 1.6 m – 1 mark · (iii)(B) Correct proportion from similar triangles – 1 mark · x = 8 m – 1 mark

Examiner tip: Match corresponding sides: the eye height corresponds to the tree height, and Riya's distance from the mirror corresponds to the tree's distance from the mirror.

38. A housing society in Thane recorded the daily water consumption (in litres) of its 50 households to plan a water-saving campaign. The data are given below:

Daily consumption (litres)	100–150	150–200	200–250	250–300	300–350	350–400
Number of households	4	6	14	12	9	5

Based on the above information, answer the following questions:

(i) Write the modal class of the data. (1 mark)

(ii) Write the median class of the data. (1 mark)

(iii) (A) Find the mean daily water consumption per household. (2 marks)

OR

(iii) (B) Find the mode of the data. (2 marks)

Answer (4 marks)

(i) The highest frequency is 14, for the class 200–250. Modal class = **200–250**.

(ii) N = 50, N/2 = 25. Cumulative frequencies: 4, 10, 24, 36, 45, 50. The first cumulative frequency greater than 25 is 36, so median class = **250–300**.

(iii) (A) Class marks x_i: 125, 175, 225, 275, 325, 375.

x_i	125	175	225	275	325	375	**Total**
f_i	4	6	14	12	9	5	50
f_ix_i	500	1050	3150	3300	2925	1875	12800

Mean = $\Sigma f_i x_i / \Sigma f_i$ = 12800/50 = **256 litres**.

OR

(iii) (B) Modal class 200–250: l = 200, h = 50, f_1 = 14, f_0 = 6, f_2 = 12.

Mode = $l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h = 200 + [(14 - 6)/(28 - 6 - 12)] \times 50 = 200 + (8/10) \times 50 = 200 + 40$ = **240 litres**.

Marking: (i) Modal class 200–250 – 1 mark · (ii) Median class 250–300 – 1 mark · (iii)(A) Table of f_ix_i ($\Sigma f_i x_i$ = 12800) – 1 mark · Mean = 256 litres – 1 mark · (iii)(B) Correct values of l, h, f_0, f_1, f_2 in the formula – 1 mark · Mode = 240 litres – 1 mark

Examiner tip: The modal class and the median class are not always the same — use the highest frequency for the modal class and N/2 with cumulative frequencies for the median class. In the mode formula, f_0 and f_2 are the frequencies of the classes just before and just after the modal class.

All the best for your exam!